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# Engineering equipment structure design method based on fuzzy optimization

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## ABSTRACT

Based on the symmetry fuzzy optimization in the structural design, constructs the iterative criterion, provides the objective function and constraints fuzzification, and give the fuzzy optimization model and solving flow chart. Takes the spindle device as the sample, according to the requirements and characteristics, build the spindle device fuzzification model, let it the smallest mass under the conditions of ensuring the strength, stiffness and other conditions. The analysis of the results showed that fuzzy optimization design method which solves the structural optimization has good development potential.

# **KEYWORDS**

Spindle device; Structural optimization; Symmetric fuzzy optimization.

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## **INTRODUCTIONS**

Optimized design is a very effective product design method which is to ensure the products with excellent performance, reducing weight and volume, reducing the cost <sup>[1-3]</sup>. Currently, the optimization method has been widely used in engineering equipment design<sup>[4-6]</sup>. An ideal device also has the smallest, lightest, most provinces and lowest cost of materials and other advantages in the reliable working conditions in addition to good power. Therefore, lightweight has become an important indicator of the advanced nature of the assessment design. With the combination of structural optimization and fuzzy mathematical theory, the structure fuzzy optimization theory is gradually paid attention<sup>[7,8]</sup>.

## THE MATHEMATICAL BASIS OF THE FUZZY OPTIMAL

The optimal solution of the fuzzy optimization problem depends on the maximum points of the membership functions of the fuzzy judgment sets. This results that the forms of the aggregate operators, the sub-objective functions and the constraints directly affect the fuzzy optimal solution. The different forms of the fuzzy judgments, the aggregate functions and the membership functions determine that the fuzzy optimal solution is not unique. This is the fuzzy optimization feature which is different from the normal optimization, it is also the advantages of the fuzzy optimization. It which the solution is not unique provides a choice of the several designs, which is in line with the characteristics of the engineering design<sup>[9]</sup>.

Based on the feature of the fuzzy optimization, conduct the analysis of the application examples<sup>[10]</sup>. creates a symmetric fuzzy optimization model, and the iterative method is used. The iterative method is suitable for solving the symmetry fuzzy optimization problems. The symmetric fuzzy optimization refers to the equally important of the objectives and constraints in the optimization problems. Thus the intersection of the fuzzy objectives sets and the fuzzy constraints has a point, It also enables the objectives and constraints to get the maximum degree of satisfaction. It uses a symmetric approach by introducing the concept of the considering the fuzzy judgment QUOTE  $\mathfrak{D} \mathfrak{D}$ , the fuzzy constraints  $\mathfrak{G}$  and the fuzzy objectives function  $\mathfrak{F}$  to be equal, denoted by  $\mathfrak{D} = \mathfrak{G} \cap \mathfrak{F}$ ,  $\cap$  is the combined operator, Membership functions can be described as a form:

## $\mu_{\widetilde{D}} \mathbf{Y} \mathbf{X} \mathbf{Y} = \mu_{\widetilde{G}} \mathbf{Y} \mathbf{X} \mathbf{Y} * \mu_{\widetilde{F}} \mathbf{Y} \mathbf{X} \mathbf{Y} , \mathbf{X} \in \mathbf{X}$ (1)

In the formula, '\* ' is the operator corresponding to  $\cap$  . Let  $\lambda$  level sets of the fuzzy constraint set  $\Im$  :

$$G_{\lambda} = \left\{ x \middle| \widetilde{G}(x) \ge \lambda_r x \in \mathbb{R} \right\}$$
(2)

The maximum of the intersection  $\mathbf{\tilde{D}} = \mathbf{\tilde{G}} \cap \mathbf{\tilde{F}}$  of the fuzzy objectives set  $\mathbf{\tilde{F}}$  and the fuzzy constraints set  $\mathbf{\tilde{G}}$ :

$$D^{*}(x^{\dagger} *) = \max P^{*}(x) = \max P(\lambda \in [0, 1]) [\lambda \wedge [\max F^{*}(x)] + (\lambda \in G_{\downarrow}\lambda)]$$
(3)

According to the above analysis, construct an iterative testing guideline. The symmetric fuzzy optimization for solving problems can be attributed to demand

$$\lambda^{*} = \max_{\lambda \in \mathcal{G}_{\lambda^{*}}} \widetilde{P}(\mathbf{x}) = \max_{\lambda \in \mathcal{G}_{\lambda^{*}}} \widetilde{P}(\mathbf{x})$$
(4)

Just seek such  $\lambda^*$ , in the level sets  $G_{\lambda^*}$  under the maximization of fuzzy objective function  $F(\mathbf{x})$ , you can get the optimal solution  $\mathbf{x}^*$ . so  $\lambda^*$  is called the optimal  $\lambda^*$ , the corresponding level sets is the optimal level set.

From the formula (4), we know:

$$\lambda^* - \max_{\lambda \in G_{\lambda^*}} \widetilde{F}(\mathbf{x}) = \mathbf{0}$$
(5)

For <sup>λ</sup><sup>•</sup> is unique, only whenλis optimal, the formula (5) was set up, otherwise it will not equal. Therefore, you can use (5) as a guideline, the process to search for the optimal solution comes down to make

$$\varepsilon^{(\mathbf{k})} = \lambda^{\mathbf{k}} - \max_{\mathbf{x} \in \mathcal{C}^{\mathbf{k}}_{\lambda}} \widetilde{\mathbf{F}}(\mathbf{x}) = 0$$
(6)

gradually approach zero. The project does not meet the requirements to get the absolute optimal solution of the formula (6), The only requirement of the formula (6) is less than the advanced given non-negative small value ' $\epsilon$ '. Therefore, the process of finding the optimal and optimal solutions can be attributed to the following conditions to make

$$|\lambda^{\mathbf{k}} - \widetilde{\mathbf{F}}(\mathbf{x}^{\mathbf{k}})| \leq \varepsilon$$
(7)

gradually met. Where, k represents the number of the iterations.

$$\widetilde{\mathbf{F}}(\mathbf{x}^{\mathbf{k}}) = \max_{\mathbf{x} \in G^{\mathbf{k}}_{\lambda}} \widetilde{\mathbf{F}}(\mathbf{x})$$
(8)

The formula (8) represents the  $\mathbf{F}(\mathbf{x})$  maximum value on the level sets  $\mathbf{G}_{\mathbf{x}}^{\mathbf{x}}$  of the k-th iteration,  $\Box' \varepsilon'$  is the advanced given convergence precision, typically the range of  $\varepsilon$  is  $\mathbf{10^{-6} \sim 10^{-6}}$ .

## FUZZY OPTIMIZATION ENGINEERING APPLICATIONS

Let the high-speed three stepped shaft of a marine engineering equipment, the material is 45 steel, the shaft middle is mounted a gear which the quality is  $\mathbf{Q} = \mathbf{10kg}$ , the shaft speed n =60r/min, each stepped section length  $\mathbf{l} = \mathbf{130mm}$ , the requirements  $\mathbf{30} \leq \mathbf{d}_1 \leq \mathbf{60}$ ,  $\mathbf{50} \leq \mathbf{d}_2 \leq \mathbf{100}$ . The density of the shaft material  $\rho = 7.85 \times 10^{-6} \text{kg/mm}^3$ , the elasticity Modulus  $\mathbf{E} = \mathbf{200GP}_a$ . Seek the design scheme to meet the requirement of the smallest mass under the conditions of dynamic stability.

(1) The stepped shaft mathematical model

1) Select the design variables.

According to the problem, select the journals as a design constant:

$$\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^{\mathrm{T}} = [\mathbf{d}_1, \mathbf{d}_2]^{\mathrm{T}}$$
(9)

2) Build the objective function. The mass is expressed as

$$W = \rho \pi l \left( 2d_2^2 + d_2^2 \right) / 4 \tag{10}$$



Figure 1 : Symmetric fuzzy optimization iterative diagram

no



Figure 2 : The stepped shaft schematic diagram

## $f(x) = \rho \pi l (2x_1^2 + x_2^2)/4$

3) Build the constraint conditions. Because of the high-speed rotation, need to ensure the stability and prevent its resonance in the critical state, should satisfy the condition:

$$\omega < 0.7\omega_{
m n}$$

where  $\omega$ : the angular velocity of the shaft,

$$\omega = \frac{2\pi\pi}{60} = 2 \times \frac{60\pi}{60} = 2\pi \text{ rad/s}$$
(13)

where  $\omega_{\mathbf{n}}$  the shaft fixed lateral frequency,

$$\omega_{\mathbf{n}} = \sqrt{\mathbf{g}/\mathbf{f}} \tag{14}$$

Where g: the acceleration due to gravity; f: the static deformation of the gravity static of the shaft

f: the static deformation of the cross-section of the shaft intermediate.

$$f = 10.67 \frac{Ql^3}{nE} \left( \frac{1}{d_1^4} + 2.\frac{38}{d_2^4} \right)$$
(15)

where E: the elastic modulus of the material, fill the relevant data and formulas in the formula (12),

$$\omega < 0.7 \sqrt{\frac{g\pi B}{10.67 Ql^3 \left(\frac{1}{d_1^4} + 2.\frac{38}{d_2^4}\right)}}$$
(16)

$$g_{1}(\mathbf{x}) = 0.\frac{0459g\pi E}{Ql^{2}\omega^{2}} - \left(\frac{1}{d_{1}^{4}} + 2.\frac{38}{d_{2}^{4}}\right) \ge 0$$
(17)

In addition, according to the structural requirements, the boundary conditions can be given as

$$30 \le d_1 \le 60$$
,  $50 \le d_2 \le 100$  (18)

4) Build the standardized mathematical model.

$$\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^{\mathsf{T}} = [\mathbf{d}_1, \mathbf{d}_2]^{\mathsf{T}}$$
(19)

$$f(x) = \rho \pi l (2x_1^2 + x_2^2)/4$$
(20)

S.t.

$$g_1(\mathbf{x}) = 0. \frac{0459 \text{gnE}}{\text{Ql}^3 \omega^2} - \left(\frac{1}{d_1^4} + 2. \frac{38}{d_2^4}\right) \ge 0$$
 (21)

(11)

(12)

$$g_2(x) = x_1 - 30 \ge 0$$
 (22)

$$g_3(\mathbf{x}) = 60 - \mathbf{x}_1 \ge 0$$

$$g_4(\mathbf{x}) = \mathbf{x}_2 - 50 \ge 0 \quad (24)$$
(23)

## $g_5(x) = 100 - x_2 \ge 0$ (25)

For simplicity, in the establishment of fuzzy optimization model,  $g_2(x)$  and  $g_4(x)$  joint are denoted  $g_2(x)$ ,  $g_8(x)$  and  $g_4(x)$  joint are denoted  $g_8(x)$ .

(2) Fuzzy optimization method for solving.

1) The standardization mathematical model transforms into the symmetric fuzzy optimization model. Need to determine the membership function of the fuzzy constraints and fuzzy objective function, Solving fuzzy optimization problems generally represents as

Find  $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T = [\mathbf{d}_1, \mathbf{d}_2]^T$ 

 $Min f(x) = \rho \pi l (2x_1^2 + x_2^2)/4$ 

S.t. 
$$b^{l} \leq \tilde{B}x \leq \tilde{D}^{u}$$
 (26)

Where  $\mathbf{B} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$ ;  $\mathbf{b}^{l} = (30, 50)^{T}$ ;  $\mathbf{b}^{u} = (60, 100)^{T}$ . Obviously the fuzzy constraints membership functions  $g_2(\mathbf{X})$ ,  $g_3(\mathbf{X})$ ,  $g_4(\mathbf{x})$ ,  $g_5(\mathbf{x})$  are the

linear functions (showed in Figure 3)



Figure 3 : The fuzzy constraints schematic

$$\mu_{i}(G_{i}i) (Bx) = \{ \blacksquare (\blacksquare (0 \qquad 0 \le (Bx)_{i}i \le b_{i}i^{\dagger}(l_{i}1) \ \Theta((Bx)_{i}i - b_{i}i^{\dagger}(l_{i}1))/(b_{i}i^{\dagger}(u_{i}1) - b_{i}i^{\dagger}(l_{i}1)) (27) \}$$

 $g_1(x)$  is nonlinear, it cannot use the linear criteria, here may take a pro-rata scaling, with respect to k = 1.2, the minimum reduction ratio 5/100=0.05, use  $h_1$  to represent.

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$$g_4(\mathbf{x}) = 1.05 \mathbf{x} \, 0. \frac{0459 \,\mathrm{gnE}}{\mathrm{Q} \mathrm{I}^3 \omega^2} - \left(\frac{1}{\mathrm{d}_1^4} + 2. \frac{38}{\mathrm{d}_2^4}\right) > 0 \tag{28}$$

Where  $(\mathbf{B}\mathbf{x})_{\mathbf{1}}$ : The i-th constraint function  $(\mathbf{k} = \mathbf{1}, \mathbf{2})$ ; obviously: i=1,  $(\mathbf{B}\mathbf{x})_{\mathbf{1}} = \mathbf{x}_{\mathbf{1}}$ ; i=2,  $(\mathbf{B}\mathbf{x})_{\mathbf{2}} = \mathbf{x}_{\mathbf{2}}$ 

> Let each linear fuzzy constraint is the same transitional interval  $h_i$ Given  $d_2$ , 3 = 5:

$$\mathbf{b}_{i}^{u} = \mathbf{b}_{i}^{l} + \mathbf{h}_{i} = \mathbf{b}_{i}^{l} + \mathbf{5} , \ i=2,3$$
(29)

$$\mathbf{b_{i}^{u_{2}}} = \left(\mathbf{b_{i}^{l_{2}}} + \mathbf{h}_{1}, \mathbf{b}_{2}^{l_{2}} + \mathbf{h}_{2}, \mathbf{b}_{3}^{l_{2}} + \mathbf{h}_{3}\right) = \left(\mathbf{1.05x\,0}, \frac{\mathbf{04\,89\,gnE}}{\mathbf{Ql^{8}\,\omega^{2}}}, \mathbf{65, 105}\right)$$
(30)

To make the objective function fuzzy, need to be pre-estimated a maximum target value at the maximum feasible region. It should the largest target value which meets all the constraints. The basic method for estimating is as followed:

$$\begin{cases} d_1 = 65\\ \frac{1}{d_1^4} + 2.\frac{38}{d_2^4} = 1.05 \times 0.\frac{0459 \text{gm E}}{\text{Ql}^3 \omega^2} \end{cases}$$
(31)

Solve  $d_2=68$ , fill  $d_1$ ,  $d_2$  into the formula (10):

$$f(\mathbf{x}) = \rho \pi l \left( 2\mathbf{x}_1^2 + \mathbf{x}_2^2 \right) / 4 = 7.85 \times 10^{-6} \times \pi \times 150 \times Y2 \times 65^2 + 68^2 Y + 4 = 12.09 \text{kg}$$
(32)

When  $d_1=25$ ,  $d_2=45$ , the objective function value is minimum.:

$$\mathbf{f}(\mathbf{x}) = \rho \pi \mathbf{l} \left( 2\mathbf{x}_1^2 + \mathbf{x}_2^2 \right) / 4 = 7.85 \times 10^{-6} \times \pi \times 150 \times Y2 \times 25^2 + 45^2 Y + 4 = 3.01 \text{kg}$$
(33)

Then construct the fuzzy membership function of the objective function :

$$\mu_{\bar{\mathbf{p}}}(\mathbf{x}) = \frac{\rho \pi i (2x_1^2 + x_2^2) / 4 - 6.01}{12.09 - 3.01}$$
(34)

Determined the fuzzy objective set after fuzzy constraints determined, the fuzzy optimization problem becomes the design space X

Find 
$$\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T = [\mathbf{d}_1, \mathbf{d}_2]^T$$
 (35)

$$\mu_{\tilde{\mathbf{D}}}(\mathbf{x}) = \mu_{\tilde{\mathbf{F}}}(\mathbf{x}) \wedge \left( \bigwedge_{i=1}^{k} \mu_{\tilde{\mathbf{G}}}(\mathbf{B}\mathbf{x}) \right)$$
(36)

2) The iterative solution.

(1) arbitrarily assigned a level value  $\lambda^{\epsilon_{12}} = 0.9$ , convergence precision  $\varepsilon = 0.001$ .

(2) solve <sup>4</sup> level set.

$$G_{\lambda}^{(1)} = \left\{ \mu_{\tilde{G}}(Bx) | \mu_{\tilde{G}}(Bx) \ge 0.9, i = 1, 2, 3, 4, 5; x > 0 \right\}$$
(37)

That:  

$$\frac{\mathbf{b}_{i}^{u} - (\mathbf{B}\mathbf{x})_{i}}{\mathbf{b}_{i}^{u} - \mathbf{b}_{i}^{i}} \ge 0.9$$

(**Bx**)<sub>i</sub> can be solved,

$$(Bx)_{i} \leq b_{i}^{u} - 0.9(b_{i}^{u} - b_{i}^{1}), i = 1, 2, 3, 4, 5$$
(39)

So the fuzzy constraint set become  $\mathbf{q}_{\mathbf{k}}^{(1)}$  level set which is the ordinary constraint.

$$\mathbf{x_1} \le \mathbf{65} - \mathbf{0.9(65} - \mathbf{60}) \tag{40}$$

$$25 + 0.9(30 - 25) \le x_4 \tag{41}$$

$$\mathbf{x_1} \le 105 - 0.9(105 - 100) \tag{42}$$

$$50 + 0.9(50 - 45) \le x_1 \tag{43}$$

$$\left(\frac{1}{d_1^4} + 2.\frac{38}{d_2^4}\right) \le 1.05 \text{ x } 0, \frac{0459 \text{ gmB}}{\text{ Ql}^3 \omega^2} - 0.9 \left(1.05 \text{ x } 0, \frac{0459 \text{ gmB}}{\text{ Ql}^3 \omega^2} - 0, \frac{0459 \text{ gmB}}{\text{ Ql}^3 \omega^2}\right)$$
(44)

(3) At  $\mathbf{G}_{\lambda}^{(1)}$  level set, solve the ordinary optimization problems. Find  $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T = [\mathbf{d}_1, \mathbf{d}_2]^T$ 

$$\mu_{\widetilde{\mathbf{D}}}(\mathbf{x}) - \mu_{\widetilde{\mathbf{f}}}(\mathbf{x}) \wedge \left( \bigwedge_{i=1}^{\widetilde{\mathbf{c}}} \mu_{\widetilde{\mathbf{G}}}(\mathbf{B}\mathbf{x}) \right)$$

S.t.

 $x_1 \le 65 - 0.9(65 - 60)$ 

25 + 0.9(80 - 25) **s** x<sub>1</sub>

 $x_1 \leq 105 - 0.9 (105 - 100)$ 

50 + 0.9(50 - 45) **s** 🗙

$$\left(\frac{1}{d_1^4} + 2.\frac{38}{d_2^4}\right) \le 1.05 x \ 0.\frac{0459 g \pi B}{Q l^3 \omega^2} - 0.9 \left(1.05 x \ 0.\frac{0459 g \pi B}{Q l^3 \omega^2} - 0.\frac{0459 g \pi B}{Q l^3 \omega^2}\right)$$

Obtain the values of  $\mathbf{x}_{1}^{(1)}, \mathbf{x}_{2}^{(1)}, \mu_{4}\mathbf{F}^{\bullet}$  ( $\mathbf{x}^{\dagger}$ ((1)) (4) calculate  $\boldsymbol{\varepsilon}^{(1)}$ .

(38)

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$$\varepsilon^{\dagger}((1)) = \lambda^{\dagger}((1)) - \mu_{4}F^{*}(x^{\dagger}((1)))$$

$$(45)$$

(5) convergence checks.

$$\begin{cases} |\varepsilon^{(1)}| > \varepsilon, & \text{No converging} \\ |\varepsilon^{(1)}| > \varepsilon, & \text{converging} \\ (6) \text{ calculate } \lambda^{(6)}. \end{cases}$$
(46)

$$\lambda_2 = \lambda^{(1)} - \alpha^{(2)} \varepsilon^{(2)} \tag{47}$$

3) Process optimization results. Noted that stepped shaft install the bearing at both ends,  $x_1$  Must end in 0 or 5. Further, the shaft diameter is preferably taken as an integer, the optimal solution can be gotten:

## $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T = [\mathbf{d}_1, \mathbf{d}_2]^T = [\mathbf{55}, \mathbf{70}]^T$

then the quality of the shaft is 10.3275kg.

#### CONCLUSIONS

This paper systematically states the fuzzy optimization method of large marine engineering equipments, analyzes the problem s of the structure symmetric fuzzy optimization. On the basis of the structural convergence algorithm, through take the high-speed spindle of the marine engineering equipment in a for the sample, gives the fuzzy process of the design variables, the objective function, the constraints particularly, build the spindle fuzzy optimization model, Solve the model, and obtain the satisfying optimization results.

As can be seen that the structure fuzzy optimization theory is one of the main direction of development of the theory of structural optimization in the future, and when combined with the finite element theory and the software ANSYS, and other methods for solving large-scale machine optimization and simulation equipment provides a practical way. The method provides a practical way to solve the whole machine optimization and simulation of the large marine engineering equipment.

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