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DOUBLE DIFFUSIVE MAGNETOHYDRODYNAMIC FREE CONVECTION AT A VERTICAL PLATE

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ABSTRACT

An isothermal permeable plate is assumed to be immersed vertically in a homogeneous viscous electrically conducting fluid containing a concentration species and a magnetic field is applied transverse to the plate. Assuming fluid viscosity to be an inverse linear function of temperature and taking into consideration Ohmic heating, double diffusive Magnetohydrodynamic (MHD) free convective flow and heat transfer at the plate is studied numerically. Assuming the flow to be two-dimensional and introducing a similarity variable, the governing equations of the problem are reduced to a set of non-linear ordinary differential equations. The equations subject to appropriate boundary conditions are solved by Nachtsheim-Swigert scheme together with a shooting technique. Taking into consideration both aiding and opposing buoyancies, the effects of magnetic field, Ohmic heating, mass diffusion and chemical reaction on different flow and heat transfer characteristics like Skin friction, Nusselt number, Sherwood number are presented and discussed. For assisting buoyancies skin friction and Nusselt number are found to diminish with increasing Schmidt number while, for opposing buoyancies, they are found to increase with increasing Schmidt number. In both the cases of assisting and opposing buoyancies, Sherwood number is an increasing function of Schmidt number.

Key words: Double diffusive free convection, MHD, Ohmic heating.

INTRODUCTION

Rossow[1] analyzed in detail, the flow of electrically conducting fluids over a flat plate in the presence of a magnetic field. Taking the transverse magnetic field to be fixed relative to the plate and also relative to the fluid, the author discussed how the magnetic field controls the fluid motion. Hartmann and Lazarus[2] pointed out that, in certain situations,

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under the influence of magnetic field, even a turbulent stream can be stabilized to an extent that the flow becomes laminar. In such situations, in fact, the stabilizing force exerted on the conducting fluid is the electric current induced by the applied magnetic field.

Gebhart and Pera[3] analysed vertical natural convection flows arising from the combined buoyancy effects of thermal and mass diffusion. Assuming concentration levels of the species to be small and using Boussinesq approximation similarity solutions were found for water and air and for different practical values of Schmidt number for both aiding and opposing buoyancy effects. Many interesting results on the flow and heat transfer characteristics and on laminar stability were presented.

It is a known fact that mass transfer effects in fluid flows play an important role in chemical processing equipment. Chen[4] studied heat and mass transfer in MHD Natural Convection flow at a permeable inclined surface with variable wall temperature and variable concentration. Suitable transformation variables were introduced and the transformed equations were solved by the use of a finite difference technique. Important results were discussed as functions of the parameters of the problem.

Hall current effect on MHD free convection flow and mass transfer at a semi-infinite vertical plate was discussed by Aboeldahab et al.[5] under the assumption of small magnetic Reynolds number. Introducing similarity variables, the governing equations were reduced into a system of non linear ordinary differential equations and solved by fourth order Runge-Kutta Method. Results were presented and discussed as functions of the parameters of the problem.

Effects of radiation and viscous dissipation on MHD transient free convection flow and mass transfer at a semi infinite vertical plate were studied by Vasu and Prasad[6]. Under the assumption of uniform heat and mass flow at the plate, the governing equations are solved numerically by the use of an implicit finite difference scheme of the Crank-Nicolson type and the results were presented and discussed as functions of the parameters of the problem.

In the present work, the authors made a numerical study of hydromagnetic free convective heat and mass transfer in electrically conducting fluids under the influence of Ohmic heating, mass diffusion and chemical reaction. The governing equations are reduced to a set of non linear ordinary differential equations by the use of a similarity transformation and the resulting equations are solved by Nachtsheim-Swigert Scheme[7] together with a shooting technique. Both the cases of aiding and opposing buoyancies is attempted for some selected values of the Prandtl number (Pr). Liquid metals are known for their high electrical conductivity and water with slight impurities is also known as a conductor of electrical energy. So numerical solutions are obtained for Pr = 7.0 (water) and for Pr = 0.064 (Lithium) for a number of values of Schmidt number and other parameters.

Formulation of the problem

A permeable vertical plate is assumed to be immersed in a homogeneous electrically conducting viscous fluid. The X-coordinate is taken vertically upwards along the length of the plate and Y-coordinate perpendicular to it. Viscosity of the fluid is assumed to vary as an inverse linear function of temperature as $\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \alpha\,(T - T_\infty)]$. Then, using Boussinesq approximation, the equations governing the boundary layer hydromagnetic free convective heat and mass transfer in fluids under the influence of a transverse magnetic field, Ohmic heating and chemical reaction are as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta\,(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho_\infty} \qquad \ldots(2)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2 \qquad \ldots(3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_1\,(C - C_\infty) \qquad \ldots(4)$$

The relevant boundary conditions are –

$$\text{at } y = 0,\ u = 0,\ v = -V_w,\ C = C_w,\ T = T_w$$

$$\text{as } y \to \infty,\ u \to 0,\ C \to C_\infty,\ T \to T_\infty \qquad \ldots(5)$$

In these equations and boundary conditions the symbols have their usual meanings. A similarity variable together with certain non dimensional variables are introduced through the relations

$$\eta = \left(\frac{G_{rx}}{4}\right)^{1/4}\frac{y}{x},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$\Psi = 4\,\nu_\infty \left(\frac{Grx}{4}\right)^{1/4} f(n)$$

Where Ψ is the stream function.

Then the governing equations (1) –(4) reduce to the non linear ordinary differential equations

$$(\theta - \theta_r)f''' - f''\theta' - 3\frac{(\theta - \theta_r)^2}{\theta_r}ff'' + 2\frac{(\theta - \theta_r)^2}{\theta_r}f'^2 - \frac{(\theta - \theta_r)^2\theta}{\theta_r}$$

$$- N\frac{(\theta - \theta_r)^2\phi}{\theta_r} + 2M\frac{(\theta - \theta_r)^2 f'}{\theta_r} = 0 \quad \ldots(6)$$

$$\theta'' + 3\,Prf\theta' + 8\,M\lambda f'^2 = 0 \quad \ldots(7)$$

$$\phi'' + 3\,Sc\,f\phi' - 2\,\gamma Sc\,\phi = 0 \quad \ldots(8)$$

The boundary conditions in terms of f, θ, ϕ are –

$$\text{at } \eta = 0,\ f = \nu_w,\ f' = 0,\ \theta = 1,\ \phi = 1$$

$$\text{as } \eta \to \infty,\ f' = 0,\ \theta = 0,\ \phi = 0 \quad \ldots(9)$$

In these equations and boundary conditions,

$\theta_r = \dfrac{-1}{\alpha\,(T_w - T_\infty)}$ is the viscosity variation parameter,

$\nu_w = \dfrac{V_w x}{3\nu_\infty}\left(\dfrac{4}{Grx}\right)^{1/4}$ is the suction/injection parameter,

$Gr_x = \dfrac{g\beta\,(T_w - T_\infty)x^3}{\nu_\infty^2}$ is the local Grashof number,

$Grc_x = \dfrac{g\beta^*\,(C_w - C_\infty)x^3}{\nu_\infty^2}$ is the Grashof number for the concentration species,

$M_x = \sqrt{\dfrac{\sigma B_0^2 x^2}{\mu_\infty}}$ is the magnetic Reynolds number,

$Pr = \dfrac{\mu_\infty c_p}{k}$ is the Prandtl number,

$\gamma = \dfrac{K_1 x^2}{\vartheta_\infty}\dfrac{1}{\sqrt{G_{rx}}}$ is the modified chemical reaction parameter,

$Sc = \dfrac{\nu_\infty}{D}$ is the Schmidt number,

$N = \dfrac{\beta^*\,(C_w - C_\infty)}{\beta\,(T_w - T_\infty)}$ is the buoyancy ratio parameter,

$M = \dfrac{M_x^2}{\sqrt{Grx}}$ is the magnetic parameter and

$\lambda = Pr.\ \dfrac{\beta\,gx}{C_p}$ is the parameter describing the effect of Ohmic heating.

Solution of the problem

The equations (6) to (8) subject to boundary conditions (9) are solved by Nachtsheim-Swigert scheme together with a shooting technique for different values of the parameters of the problem. For the fluids under consideration, viscosity variation with temperature is minute and so computations are made only for a single value of θ_r ($\theta_r = 500$). Computations are also made for a single value of ν_w ($\nu_w = 0.1$).

RESULTS AND DISCUSSION

It may be noted that the parameters of the problem are θ_r - the viscosity variation parameter, ν_w - the suction parameter, P_r - the Prandtl number, γ - the chemical reaction parameter, S_c - the Schmidt number, N - the buoyancy ratio parameter, M - the magnetic parameter and λ - the parameter describing the effect Ohmic heating. In this report, solutions are presented for a large value of θ_r ($\theta_r = 500$), a single value of ν_w ($\nu_w = 0.1$) and some values of other parameters. Since the fluids under consideration are expected to be conductors of electric current two values are assigned to prandtl number, namely 7.00 and 0.064, which correspond to water and lithium, respectively. Lithium being a liquid metal, is a good conductor of electric energy and water with slight impurities is also a conductor of electric energy. The characteristics of practical interest are the skin friction, the wall heat transfer rate or the Nusselt number and the Shearwood number and so results pertaining to those characteristics only are presented in this report. The results are presented in two cases – Case-(i) Pr = 7.0 (Water) and Case-(ii) Pr = 0.064 (Lithium).

Case (i): Pr=7.0 (Water)

Variations in skin friction ‘f′′ (0)′ with the Shmidt number (Sc) are presented in Fig. 1 for a positive value of N, i.e., for aiding buoyancies. The skin friction is observed to diminish with increasing values of the Schmidt number and approach a constant value asymptotically as Sc further increases. The decreasing tendency in skin friction is quite sharp for small values of Sc (Sc < 10) and not at all considerable for large values of Sc (Sc > 10). Also, for any value of the Schmidt number, the skin friction diminishes with increasing values of M, or with increasing intensity of the magnetic field.

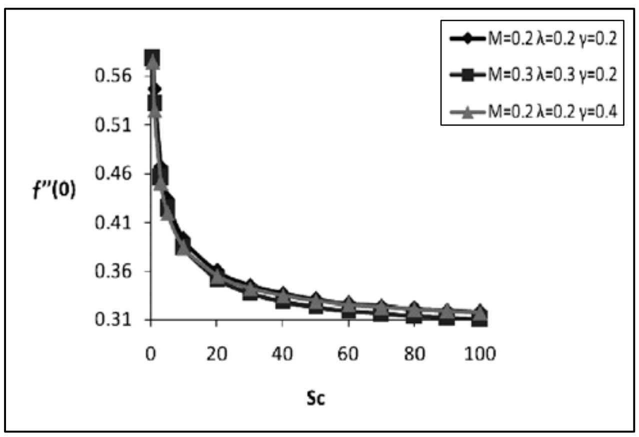

Fig. 1: Plots of skin friction versus the Schmidt number for Pr = 7.0, N = 0.5, ν_w = 0.1, θ_r = 500

Variations in the Nusselt number or the wall heat transfer rate ‘-θ′ (0)′ with the Schmidt number (Sc) are presented in Fig. 2 for aiding buoyancies. The variations with either Sc or M are exactly similar to those of the skin friction with the respective parameters.

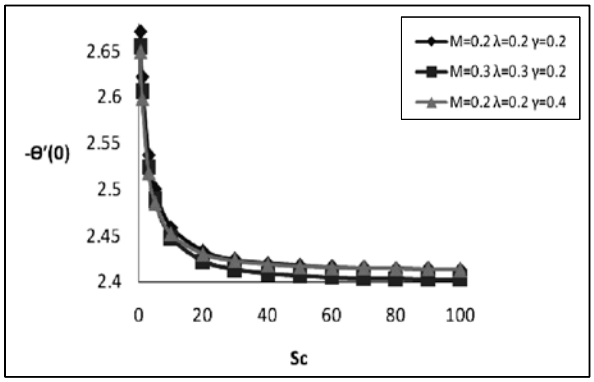

Fig. 2: Plots of Nusselt number versus Schmidt number for $Pr = 7.0$, $N = 0.5$, $\nu_w = 0.1$, $\theta_r = 500$

Variations in the Sherwood number '-ϕ′ (0)′ with the Schmidt number (Sc) are presented in Fig. 3 for aiding buoyancies. From the figure, Sherwood number can be seen to be a linearly increasing function of the Schmidt number. Like the skin friction and the Nusselt number, the Sherwood number also diminishes with increasing intensity of the magnetic field. However, Sherwood number increases with increasing values of γ, when there is enhanced chemical reaction.

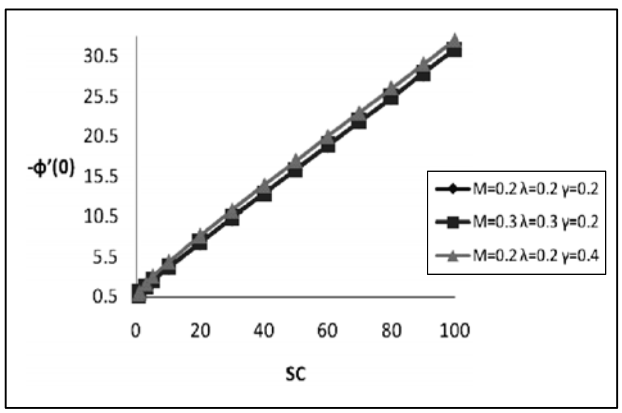

Fig. 3: Plots of Sherwood number versus the Schmidt number for for $Pr = 7.0$, $N = 0.5$, $\nu_w = 0.1$, $\theta_r = 500$

Plots of the skin friction 'f″ (0)′, the Nusselt number '-θ′ (0)′ and the Sherwood number '-ϕ′ (0)′ versus the Schmidt number Sc for a negative values of N, i.e., for opposing buoyancies are presented in Figs. 4, 5 and 6, respectively. Unlike in the case of aiding flows, in the opposing flow case, the skin friction as well as the Nusselt number increase with increasing values of the Schmidt number and approach a constant value asymptotically for further increasing values of Sc. Also the diminishing nature of f′′ (0) and '-θ′ (0)′ with Sc is striking only for small values of Sc (Sc < 10). Further for all values of Sc, the skin friction diminishes with increasing intensity of the magnetic field for both aiding and opposing buoyancies. But for opposing buoyancies, the Nusselt number increases with increasing values of M for small values of Sc (Sc = 1) and decrease with M for all other large values of Sc. However, it is interesting to note that the Sherwood number increases linearly with increasing values of Sc for both aiding and opposing buoyancies. Skin friction, Nusselt number as well as Sherwood number can be found to assume increasing values with increasing values of the chemical reaction of parameter (γ) for all values of the Schmidt number (Sc). This indicates that when there is enhanced chemical reaction, there can be enhanced flow, heat transfer and mass transfer.

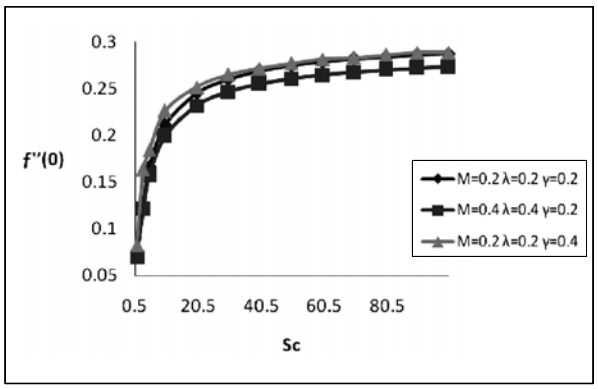

Fig. 4: Plots of Skin friction versus the Schmidt number for $Pr = 7.0$, $N = - 0.5$, $v_w = 0.1$, $\theta_r = 500$

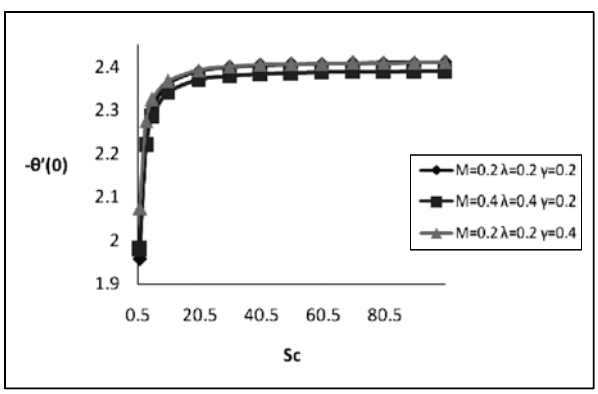

Fig. 5: Plots of Nusselt number versus the Schmidt number for $Pr = 7.0$, $N = - 0.5$, $v_w = 0.1$, $\theta_r = 500$

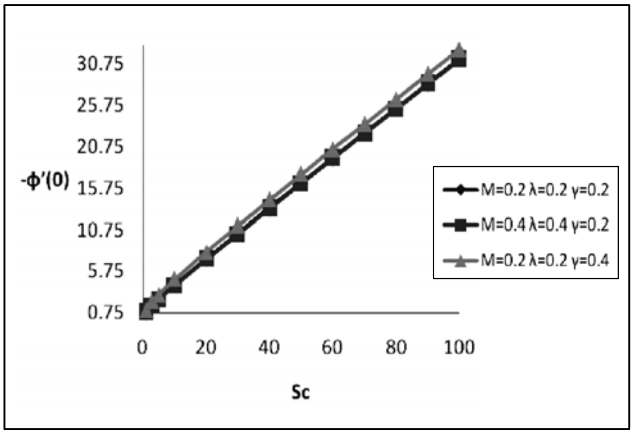

Fig. 6: Plots of Sherwood number versus Schmidt number for $Pr = 7.0$, $N = -0.5$, $\nu_w = 0.1$, $\theta_r = 500$

In Figs. 7 to 8, variations in the skin friction, Nusselt number and Sherwood number respectively for opposing and assisting buoyancies are shown. Both skin friction and Nusselt number can be found to increase with increasing values of N but variations in Nusselt number are relatively smaller as compared to those of skin friction. The behaviours of both skin friction and Nusselt number for opposing buoyancies are quite opposite to those for assisting buoyancies except for the magnetic parameter M. (e.g., for opposing buoyancies, $f''(0)$ increases with increasing Schmidt number Sc while for aiding buoyancies $f''(0)$ decreases with increasing Sc. However, in case of both opposing and assisting buoyancies, $f''(0)$ decreases with increasing magnetic parameter M.).

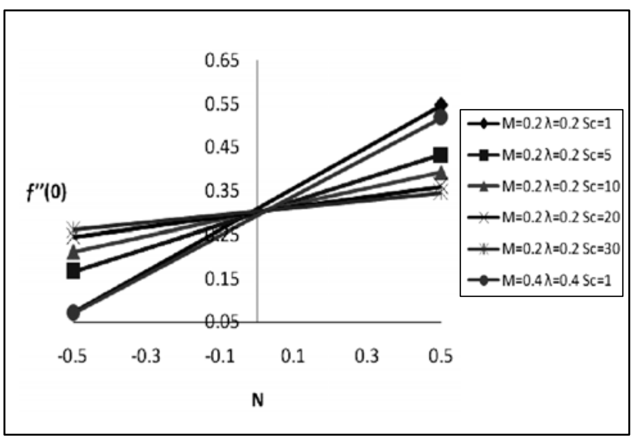

Fig. 7: Variations in skin friction with buoyancy ratio parameter N for $Pr = 7.0$, $\nu_w = 0.1$, $\theta_r = 500$, $\gamma = 0.2$

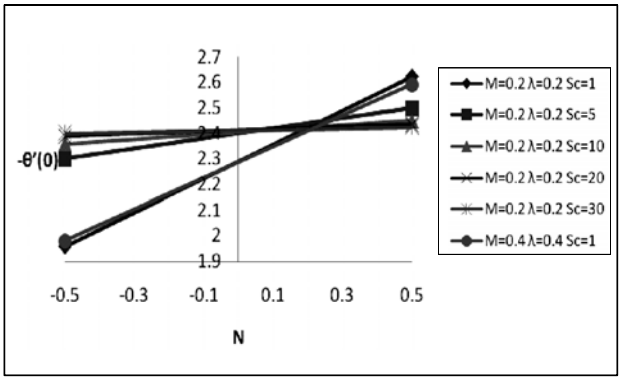

Fig. 8: Variations in Nusselt number with buoyancy ratio parameter N for $Pr = 7.0$, $\nu_w = 0.1$, $\theta_r = 500$, $\gamma = 0.2$

From plots of Fig. 9, one may notice that there are minute changes in Sherwood number with the buoyancy ratio parameter N. As N changes from negative to positive, Sherwood number increases, the change being relatively more for small values of Sc and much less for large values of Sc (≥ 20). Sherwood number assumes smaller values for opposing buoyancies and assumes relatively larger values for assisting buoyancies. Increasing values of magnetic parameter decrease Sherwood number in the assisting buoyancies case and increase sherwood number in the opposing buoyancies case.

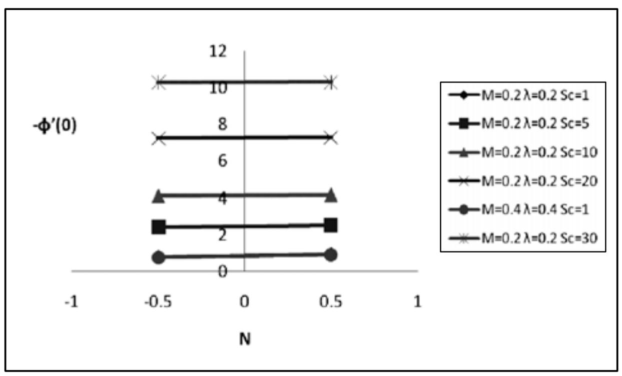

Fig. 9: Variations of Sherwood number with Buoyancy ratio parameter N for $Pr = 7.0$, $\nu_w = 0.1$, $\theta_r = 500$, $\gamma = 0.2$

Case (ii): Pr=0.064 (Lithium)

Variations in skin friction, Nussselt number and Sherwood number are presented in Figs. 10, 11 and 12, respectively for a positive value of N (i.e., for assisting buoyancies) and in Figs. 13, 14 and 15, respectively for a negative value of N (i.e., for opposing buoyancies).

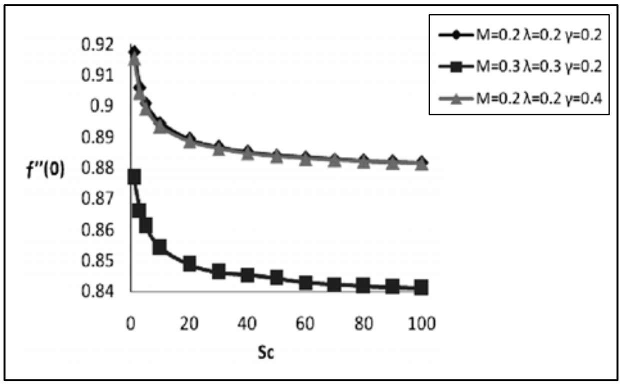

Fig. 10: Plots of skin friction versus the Schmidt number for Pr = 0.064, N = 0.1, λ = 0.2, ν_w = 0.1, θ_r = 500

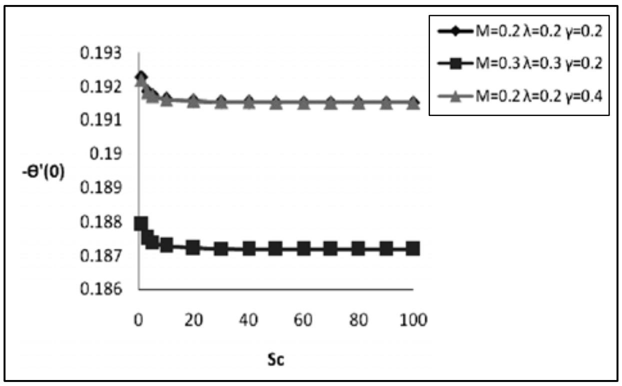

Fig. 11: Plots of Nusselt number versus Schmidt number for Pr = 0.064, N = 0.1, λ = 0.2, ν_w = 0.1, θ_r = 500

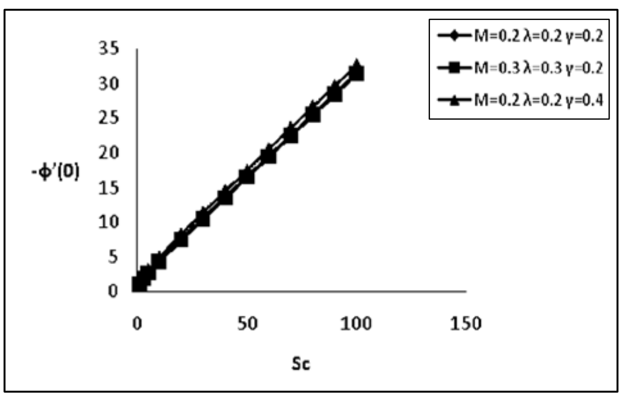

Fig. 12: Plots of Sherwood number versus the Schmidt number for Pr = 0.064, N = 0.1, λ = 0.2, ν_w = 0.1, θ_r = 500

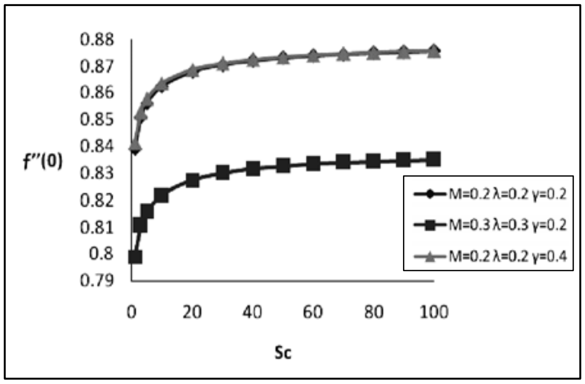

Fig. 13: Plots of skin friction versus the Schmidt number for
Pr = 0.064, N = 0.1, ν_w = 0.1, θ_r = 500

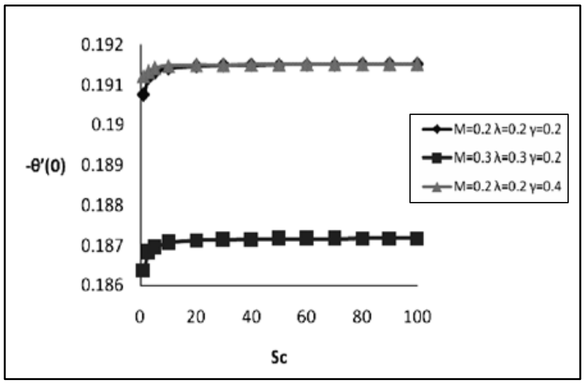

Fig. 14: Plots of Nusselt number versus Schmidt number for
Pr = 0.064, N = 0.1, ν_w = 0.1, θ_r = 500

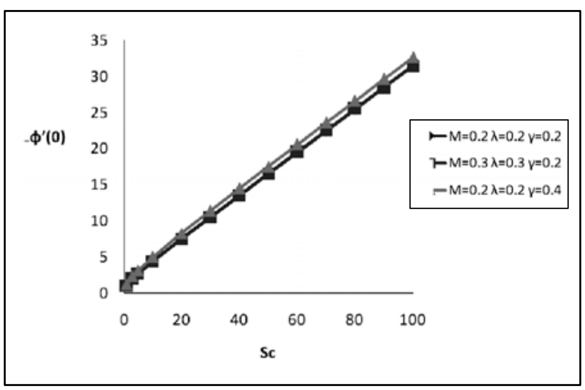

Fig. 15: Plots of Sherwood number versus the Schmidt number for
Pr = 0.064, N = -0.1, ν_w = 0.1, θ_r = 500

Comparing these figures for lithium (Pr = 0.064) with the corresponding figures for water (Pr = 7.0), one may notice that the variations in the flow, heat and mass transfer characteristics are similar except for magnitudes. That is, in the case of lithium also, in case of assisting buoyancies, skin friction and Nussult number diminish with the increasing values of Sc. Further Sherwood number increases linearly with increasing values of Sc.

In Figs. 16, 17 and 18, variations in skin friction, Nussult number and Sherwood number with the buoyancy ratio parameter N are presented. Comparing these Figs. 16 to 18 for lithium with the corresponding Figs. 7 to 9 for water, one can notice that the variations in the flow, heat and mass transfer characteristics for both the fluids under consideration are similar to one another.

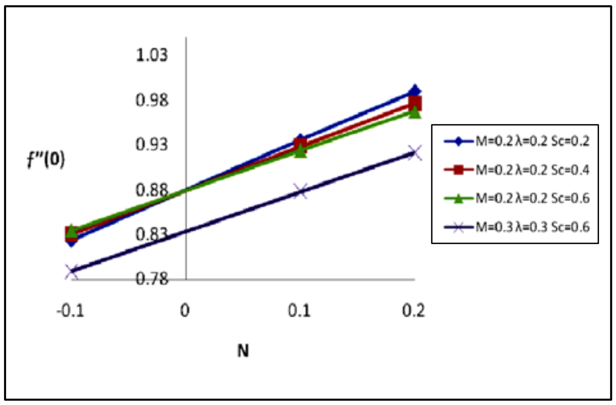

Fig. 16: Variations in skin friction with buoyancy ratio parameter N for $Pr = 0.064$, $\nu_w = 0.1$, $\theta_r = 500$, $\gamma = 0.2$

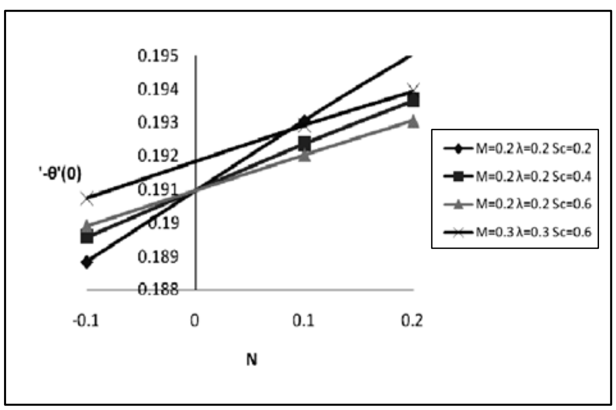

Fig. 17: Variations in Nusselt number with buoyancy ratio parameter N for $Pr = 0.064$, $\nu_w = 0.1$, $\theta_r = 500$, $\gamma = 0.2$

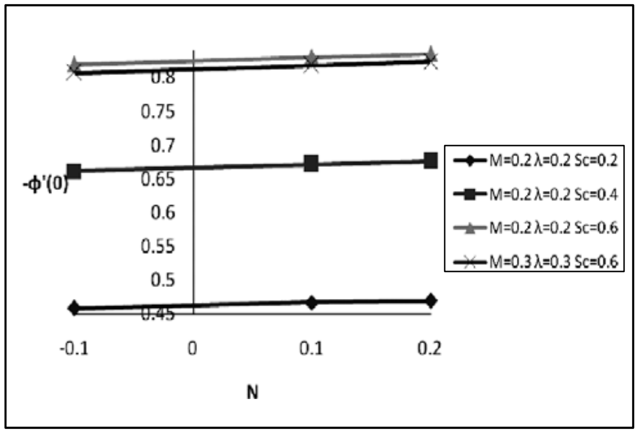

Fig. 18: Variations of Sherwood number with buoyancy ratio parameter N for Pr = 0.064, ν_w = 0.1, θ_r = 500, γ = 0.2

Like for water, for lithium also, skin friction as well as Nussult number assume smaller numerical values in the opposing buoyancies case and assume relatively larger numerical values in the assisting buoyancies case. Variations in the Sherwood number with buoyancy ratio parameter (N) are minute in the case of water and so also with the Schmidt number (Sc).

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