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Differential equation-based promote basketball field-goal percentage kinematics correlation factor research

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ABSTRACT

In basketball game, it usually shots a ball that decide winning or losing in the last minutes of game, so improve player field-goal percentage is very important. The paper carries out kinematics analysis of basketball motion trajectory in free throw and free shot two cases, and combine with mathematical differential method, it gets basketball goal rate is related to basketball release height, speed and angle. With the help of MATLAB software, it makes analysis of data in case of free throwing, it gets that promote basketball release angle accuracy can more easily promote goal rate. Analysis of free shot, it is clear that basketball best release angle is 42.7925°~62.4099°. And basketball release angle gets higher; speed gets bigger that between 8~9 m/s, it is easier to make the goal. Therefore, it suggests player to control basketball release angle to promote goal rate, control angle is between 42.7925°~62.4099°.

KEYWORDS

Basketball goal rate; Kinematics; Mathematics differential; MATLAB software; Biomechanics.

INTRODUCTION

Basketball is originated in December, 1891, is created by Doctor James Naismith, a teacher in Christian young students association training school. In the beginning, only fixed two basket that used for loading peaches in the two terminals of bleach in the building of school gym, used rugby as competition tool, and threw into basket. Subsequently basketball's basket was changed into iron-made circle and hang with cables that formed into current hoop. In order to perfect the newly developed movement, Doctor Naismith formulated 18 rules in 1892. Later, with rules revising and increasing, basketball has been gradually perfected, finally participation members have been reduced to five people, and competition divides into first half and second half, which becomes prototype of present basketball.

In 1904 the third Olympic Games, it is the first time that it has carried out basketball exhibition match. In 1908, America formulated national unified basketball rules, and used several kinds of words to publish in the world, which let people to have further understanding on basketball. In 1936 the 11th Olympic Games, it listed men's basketball game as formal competition event, and unified world basketball contest rules. After that, basketball rules are constantly revised, and gradually formed into modern basketball complete system. Basketball was introduced to China by Tianjin Chinese young men's Christian association around 1896, subsequently it spread to Beijing, Shanghai. In 1910 national Games, it firstly hosted men's basketball exhibition match, and subsequently national universities and high schools basketball activities have gradually been organized.

Up to now, basketball development has already become one of people's favorite sports events. World hosts numerous basketball games every year, from which the most famous and largest effective one is American NBA, and it also has European basketball league match, Stankovic Cup, world championship and so on. Every year events attract people's attentions, and NBA in May, this year, it can be supposed as another world hot events in May. Similarly, China also has its own basketball association that is Chinese basketball association and calls CBA for short. It is unique legal organization that represents China to attend international basketball joint association and Asian basketball joint association. Therefore, it can also see that Chinese emphasis on basketball.

Basketball, no matter men or women, young or old, it often can see people in different age stages in city basketball court and combat in the same basketball court. Basketball is a sport event that integrates running, jumping, shooting and other multiple motions. By participating basketball, it can improve people physical quality. Basketball is a kind of creative activity that has certain techniques and tactics, and also contains individual different styles. It needs people to play games with their own ways and adjust at times according to field status. And meanwhile, it also trains people observation ability and reaction ability. And basketball is a kind of team sports, in competition process, it needs athlete coordination and cooperation, and meanwhile it also trains people cooperative ability.

Basketball games, no matter in world top games, or in city basketball court, the most important point is always unchanged that is to make the hoop. Though players' cooperation is also indispensible point, with players and opponents changes, players cooperation is constantly changing, there is no formulary. However, shooting in different fields are unchanged. And in basketball games different teams' shooting overall strength gap is not very big, often one score decides winning or losing, especially more common in world top events. If it can improve every player field-goal percentage a little, then team entirely can get bigger advancement. Thereupon, it is clear that field-goal percentage when shoot is particularly important to player and team. Therefore, the paper studies on basketball shooting so as to promote basketball field-goal percentage.

PROBLEM RESEARCH

Use kinematics to analyze correlation factors

To promote basketball field-goal percentage, firstly it needs to find out field-goal percentage correlation factors. In 2010, international basketball federation made new changes to basketball field again. Except for American university basketball N.C.A.A game stadium and American new professional basketball NBA field accidents, other basketball fields' game fields have been changed into field of new situations, as Figure 1 shows.

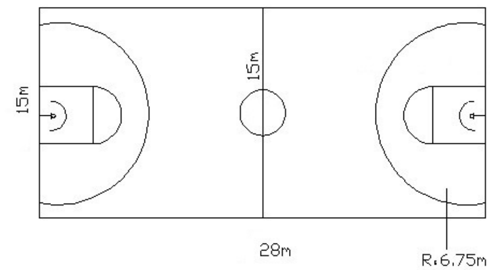

Figure 1 : New basketball field schematic diagram

International basketball federation regulated: hoop center height (use symbol H to represent) is 3.05m; hoop diameter (use symbol D to represent); basketball standard diameter (use symbol d to represent) is 24.6m;In order to let computation to be more regular, discuss player service cases. Distance that free throw line from basketball court bottom line is 5.80. And distance between basketball central vertical line and ground intersection point from end line inner edge point is nearly 1.4m that free throw line and hoop distance is 4.6m. When player makes free throwing, basketball makes parabola motion. Schematic diagram when player carries out free throwing is as Figure 2 shows.

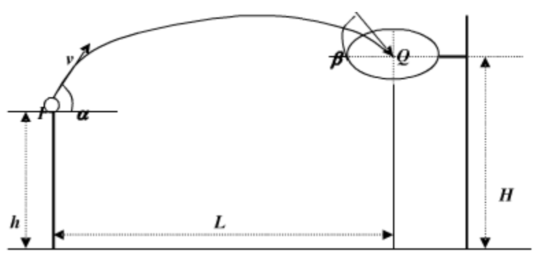

Figure 2 : Player free throwing schematic diagram

In Figure 2, P point is free throwing point, Q point is hoop center. α and β respectively shows ball release angle and incident angle when arriving at hoop. Ball initial speed is v .

Apply kinematics cast motion knowledge, analyze basketball motion features: Basketball goal ways are mainly swishing and rebounding two types. In actual free throwing conditions, according to experiences, it is clear that most athletes will select swishing way, swishing has directly facing center and eccentric two types. Firstly discuss basketball directly faces to center such case.
Establish motion model, in horizontal direction:

$$\mathbf{p(A) = \frac{v(A)}{v(s)} = \pi(\frac{D-d}{2D})^2} \tag{1}$$

In vertical direction:

$$\mathbf{y = (v\sin\alpha)t - \frac{1}{2}gt^2} \tag{2}$$

Among them, t is basketball air movement time.
Utilize formula (1) and formula (2) can eliminate t, it gets:

$$y = x\tan\alpha - \frac{gx^2}{2y^2\cos^2\alpha} \tag{3}$$

When make free throwing, according to experiences, most athletes will select directly shooting and try to avoid rebound shot. So we only consider direct shooting instant status. When shooting directly, it has $y = H - h$, now $x = L$. Input formula into formula (3), it can get:

$$\tan\alpha = \frac{v^2}{gL}[1 \pm \sqrt{\frac{2g}{v^2}(H - h + \frac{gL^2}{2v^2})}] \tag{4}$$

By the formula, it is clear that field-goal percentage is mainly up to h, v, α the three points. Basketball player's heights are certain, so field-goal percentage is mainly up to basketball release speed v, and basketball release angle α.

Above formula should meet condition:

$$\frac{2g}{v^2}(H - h + \frac{gL^2}{2v^2}) \ge 0 \tag{5}$$

It solves:

$$v^2 \ge g[H - h + \sqrt{L^2 + (H - h)^2}] \tag{6}$$

Then it can see that for every height h, it corresponds to two speed values, according to subject minimum speed value meets requirement. Then it can calculate minimum speed value that meets requirements.

According to formula (3), it can get:

$$\tan\beta = -\frac{dy}{dx}\Big|x = L \tag{7}$$

Simultaneous formula (3) and can get:

$$\tan\beta = \tan\alpha - \frac{2(H - h)}{L} \tag{8}$$

Basketball incident angle decides basketball can enter or not. In this way, it can get basketball center shooting moment incidence relations with speed and angle by formula (6) and (8).

When basketball eccentric incidents:

Set when basketball incidents into hoop very edge, its position difference with hoop dead center is Δx, then:

$$\Delta x = \frac{D}{2} - \frac{d}{2\sin\beta} \tag{9}$$

Now, basketball incidents to hoop instant position and player position distance changes into $L + \Delta x$. Input the condition into formula (3), it gets:

$$x^2\frac{g}{2v^2\cos^2\alpha} - x\tan\alpha + H - h = 0 \tag{10}$$

And utilize mathematical differential knowledge, regard v as a fixed quantity, and then basketball release angle can also be defined. Make differential on above formula. It can get:

$$\frac{dx}{d\alpha} = \frac{Lv^2 - gL^2 \tan\alpha}{gL - v^2 \sin\alpha\cos\alpha} \quad (11)$$

Sort out and get:

$$\Delta\alpha = \frac{gL - v^2 \sin\alpha\cos\alpha}{Lv^2 - gL^2 \tan\alpha}\Delta x \quad (12)$$

According to α value, combine with formula (6) and (8), it can get:

$$\Delta v = \frac{gL - v^2 \sin\alpha\cos\alpha}{gL^2} v\Delta x \quad (13)$$

According to above formula, it is clear whether basketball makes the hoop is related to basketball release angle and speed.

In basketball court, player shooting ways and distances have varieties of types. In competition, player shooting distance is normally within 6.75m. And inside 1.25m is charging zone, not considering the zone shooting status. The whole process doesn't consider the case of dunking.

When shooting zone is between 1.25m and 6.75m, shooting ways are relatively changeable, it has jump shot, tear drop and set shot so on. But basketball still makes cast motion, now L and h values are changing. Regarding basketball motion trajectory as a line in plane, then basketball motion trajectory will get through point $(L-\Delta x, H-h)$ and $(L+\Delta x, H-h)$.

Then according to formula (3), it can get basketball motion trajectory O equation is:

$$y = x\tan\alpha - \frac{(L-\Delta x)\tan\alpha - H + h}{(L-\Delta x)^2} \quad (14)$$

$$y = x\tan\alpha - \frac{(L+\Delta x)\tan\alpha - H + h}{(L+\Delta x)^2} \quad (15)$$

Then straight line OP_1 equation is:

$$y = \frac{H-h}{L-\Delta x}x \quad (16)$$

$$y = \frac{H-h}{L+\Delta x}x \quad (17)$$

Basketball field-goal percentage can use basketball maximum deviation two straight lines and hoop surrounded graphics area to represent, then:

$$A_{OP_1} = \int_0^{L-\Delta x} [x\tan\alpha - \frac{(L-\Delta x)\tan\alpha - H + h}{(L-\Delta x)^2}x^2 - \frac{H-h}{L-\Delta x}x]dx \quad (18)$$

$$A_{OP_1} = \int_0^{L+\Delta x} [x\tan\alpha - \frac{(L+\Delta x)\tan\alpha - H + h}{(L+\Delta x)^2}x^2 - \frac{H-h}{L+\Delta x}x]dx \quad (19)$$

$$\mathbf{A_{OP_1P_2} = \frac{1}{2}[2\Delta x(H-h)]} \quad (20)$$

$$A_a = A_{OP_2} - A_{OP_1} - A_{OP_1P_2} = \frac{2}{3}L\Delta x \tan\alpha - \frac{4}{3}\Delta x(H-h) \quad (21)$$

By formula (21), it is clear that $\tan\alpha$ gets bigger, field-goal percentage will get bigger. And α value is defined by speed v. Assume curve gets through point $(L, H-h)$, then input the point into formula, it can get:

$$\tan\alpha = \frac{1}{gL}(v^2 - \sqrt{v^4 - 2v^2(H-h)g - g^2L^2}) \quad (22)$$

So v needs to meet conditions:

$$v^4 - 2v^2(H-h)g - g^2L^2 \geq 0 \quad (23)$$

Make differential on $\tan\alpha$, and then it meets:

$$\frac{d\tan\alpha}{d(v^2)} = \frac{\sqrt{v^4 - 2v^2(H-h)g - g^2s^{t\theta}} - v^2 + (H-h)g}{gs\sqrt{v^4 - 2v^2(H-h)g - g^2s^2}} \leq 0 \quad (24)$$

It is clear that $\tan\alpha$ is decreasing function. According to formula analysis, it is clear that when v is smallest, $\tan\alpha$ is the largest. According to formula (6), it is clear that $\tan\alpha$ maximum value is:

$$MAX\tan\alpha = \frac{H-h}{L} + \sqrt{(\frac{H-h}{L})^2 + 1} \quad (25)$$

$$\alpha = \arctan(\frac{H-h}{L} + \sqrt{(\frac{H-h}{L})^2 + 1}) \quad (26)$$

In topic, L range is $(L-\Delta x, L+\Delta x)$, here assume $R = \Delta x$. Then it can get:

$$\arctan[\sqrt{(\frac{H-h}{L+R})^2 + 1}] \leq \alpha \leq \arctan[\frac{H-R}{L-R}\sqrt{(\frac{H-h}{L+R})^2 + 1}] \quad (27)$$

According to the formula, it can get different positions and release height to calculate basketball shooting incident angle range and basketball field-goal percentage main influence factors.

In basketball motion process, it will suffer air resistance effect. Except for resistance, basketball will suffer gravity of earth effects. So resistance has relative big influence on vertical speed than horizontal speed. Resistance coefficient is in direct proportion to speed, set proportion coefficient is k, then basketball horizontal direction sports differential equation is:

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} = 0 \quad (28)$$

$$\frac{dx}{dt}\Big|_{t=0} = v\cos\alpha \quad (29)$$

Use mathematical analysis method, it can solve:

$$x(t) = \frac{1-e^{-kt}}{k} v\cos\alpha \tag{30}$$

By Taylor formula simplifying above formula, it can get:

$$\mathbf{x(t) = v\cos\alpha t - \frac{kv\cos\alpha t^2}{2}} \tag{31}$$

When player makes three-point shot, both L value and α value appear changes. According to formula (2), it can get:
Horizontal direction:

$$\mathbf{x(t) = v\cos\alpha t - \frac{kv\cos\alpha t^2}{2}} \tag{32}$$

Vertical direction:

$$\mathbf{y(t) = v\sin\alpha t - \frac{1}{2}gt^2} \tag{33}$$

When basketball just shots on sphere center, then basketball motion trajectory passes through point $(L, H-h)$. When player stands three-point line to shoot, distance between basketball and hoop dead center is 6.75m, now L value is 6.75m. Equation changes into:

$$\mathbf{L = v\cos\alpha t - \frac{kv\cos\alpha t^2}{2}} \tag{34}$$

$$\mathbf{H - h = v\sin\alpha t - \frac{1}{2}gt^2} \tag{35}$$

Simultaneous two formulas and simplify, firstly eliminate time t and get:

$$\mathbf{H - h = \frac{Lg\sin\alpha - (H-h)kv\sin\alpha\cos\alpha}{\cos\alpha(g - vk\sin\alpha)} - \frac{g[Lg - (H-h)kv\cos\alpha]^2}{2v^2\cos^2\alpha(g - vk\sin\alpha)^2}} \tag{36}$$

The equation represents basketball shoots from player hand to end with basketball entering into hoop, basketball air motion trajectory. From basketball motion trajectory, it is clear whether basketball makes the hoop or not. By formula, it is clear basketball final goals or not is related to basketball release height, speed and angle.

Data calculation

When player makes free throwing, basketball swishes. When player shoots, body will squat, arms bend and put basketball in head nearly 10cm height position. Basketball player height generally is between 1.9m and 2.2m. To comprehensive consider, take basketball release height between 1.8m and 2.1m.
Input data into formula (6), with the help of MATLAB software, calculate minimum speed value. It gets basketball release speed minimum value is as TABLE 1 shows.

TABLE 1 : When making free throwing, basketball release height h corresponds to minimum speed v

Basketball release height h(m)	Minimum speed v(m/s)
1.8	7.6789
1.9	7.5985
2.0	7.5186
2.1	7.4392

By above TABLE 1, it is clear that shooting minimum speed will reduce with basketball release height increasing. It conforms to physical knowledge and minimum speed is less than 8m/s, which conforms to speed hypothesis in the paper. And in actual shooting, player is hard to control basketball speed at minimum speed instant to shoot. Basketball release speed is generally between 8~9m/s, so here it takes basketball release speed as smaller value 8m/s.

When basketball eccentric incidents, according to formula (26), with the help of MATLAB software, input data when free throwing goals, it gets when basketball free shoots, basketball release angle values can refer to TABLE 2.

TABLE 2 : Release angle

Release speed V(m/s)	Height h(m)	Release angle α
8	1.8	62.4099
8	1.9	63.1174
8	2.0	63.7281
8	2.1	64.4121

According to formula (12)and(13), it can get angle and speed deviation value. With the help of MATLAB software, it can get Δv and $\Delta\alpha$ value as TABLE 3 shows.

TABLE 3: Δv and $\Delta\alpha$ value

h(m)	α	V(m/s)	Δv	Δα
1.8	62.4099	8	0.0528	-0.7562
2.0	64.4121	8	0.0621	-0.7174

By above TABLE 3, it is clear speeds minimum deviation is less than angle. It illustrates when shooting and making the hoop, basketball speed accuracy is required to be higher than angle, it is clear that improve player basketball release angle is more easily to improve player goal rate. In daily training, it has big requirement on speed, player basketball release speed are generally required to be big, player basketball release speed is generally required to be bigger than 8m/s, and meanwhile ensure correct release angle.

To different release heights, when basketball release speed is at 8~9m/s, basketball release angle and basketball shooting angle change range is as TABLE 4 shows.

TABLE 4: Basketball release angle and basketball shooting angle change range

V(m/s)	h(m)	α_1	α_2	β_1	β_2
8	1.8	62.4099	42.7925	53.8763	20.9213
	1.9	63.1174	40.9188	55.8206	20.1431
	2.0	63.7281	39.1301	57.4941	19.6478
	2.1	64.2671	37.4017	58.9615	19.6478

8.5	1.8	67.6975	37.5049	62.1740	12.6252
	1.9	68.0288	36.0075	63.1884	12.7753
	2.0	68.3367	34.5214	64.1179	13.0240
	2.1	68.6244	33.0444	64.9729	13.3583
9	1.8	71.0697	34.1324	67.1426	7.6551
	1.9	71.2749	32.7614	67.7974	8.1663
	2.0	71.4701	31.3881	68.4098	8.7321
	2.1	71.6561	30.0127	68.9841	9.3472

CONCLUSION

By first model analyzing shooting instant basketball trajectory, it is clear basketball field-goal percentage is related to basketball release height, speed and angle. According to problems to research on free throwing conditions, it finds that player shooting minimum speed is between 7~8 m/s. When basketball release speed is certain, basketball release height gets bigger; basketball release angle will get bigger. And when shooting, to let basketball to enter into hoop, it is required that basketball release speed deviation to be smaller than angle. In this way when player shoots, basketball control strength is certain that basketball release speed is certain, it needs players to better control basketball release height and angle so that promote basketball goal rate.

When make free shooting, player generally shoots in the range of 1.25~6.75m. By TABLE 3 data, it is clear that when make free shooting, basketball release speed gets bigger, release heights get higher, then during goal, basketball release angle range gets bigger, basketball incident angle range also gets bigger, now it is most easily to make the hoop. Basketball minimum release angle range is 42.7925°~62.4099°, now basketball release speed is 8m/s, release height is 1.8m, basketball shooting in hoop instant angle is 20.9213°~53.8763°. When release angle is in the interval range, basketball release speed and height increase, basketball still can shoot in. Therefore basketball shooting best angle is 42.7925°~62.4099°.

In addition, basketball court competition situations will change; it also suggests players to increase physical quality training so as to cope with urgency occurred in the field.

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