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C.C.Onuchukwu[1*], A.C.Ezeribe[2]

[1]Department of Industrial Physics, Anambra State University, Uli, (NIGERIA)
[2]Department of Physics/Industrial Physics, Nnamdi Azikiwe University, Awka, (NIGERIA)
E-mail: onuchukwu71chika@gmail.com

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*Corresponding author's Name & Address

C.C.Onuchukwu
Department of Industrial Physics, Anambra State University, Uli, (NIGERIA)
E-mail: onuchukwu71chika@gmail.com

Constraining the Hubble parameter using distance modulus – Redshift relation

Abstract

Using the relation between distance modulus ($m - M$), redshift (z) and deceleration parameter (q_0), obtained using Friedman-Robertson-Walker (FRW) metric, we constrain the Hubble parameter (h). Using the mean and the median values of our data obtained from NASA Extragalactic Database (NED), the value of Hubble parameter we estimated is 0.66 ± 0.11 and 0.84 ± 0.11 for $q_0 = -1$ and $q_0 = 0$ respectively. In general, using the minimum and the maximum values of the parameter in our data, the value of Hubble parameter we estimated lies in the range $0.6 \pm 0.2 \leq h \leq 2.2 \pm 0.7$, with an average value of $h = 0.67 \pm 0.22 - 0.96\ 0.29$, giving an average age for the universe as $11.4 \pm 2.4 - 14.3 \pm 2.2$ *Gyr*.

Key Words

Cosmology - Miscellaneous; Astronomical database - Miscellaneous; Method - Statistical; Data Analysis.

INTRODUCTION

The universe is all of space, time, matter and energy that exist. The ultimate fate of the universe is determined, through its gravity, thus, the amount of matter/energy in the universe is therefore a considerable importance in cosmology. The Wilkinson Microwave Anisotropy Probe (WMAP) in collaboration with National Aeronautics and Space Admiration (NASA) in one of their mission estimated that the universe comprises of about 4.6% of visible matter, 24% of matter with gravity but do not emit observable light (the dark matter) and about 71.4% was attributed to dark energy (possibly anti-gravity) that may be responsible for driving the acceleration of the observed expansion of the universe[20]. The universe has been observed to be expanding, first suggested by Einstein in his general theory of relativity and observed by Hubble[17,21,29,30].

Amongst the different models proposed to explain the evolution of the universe, the Big Bang Theory is presently the most acceptable model that described most of the observational features in the evolution of the universe. It explained the universe to have started from an extremely hot dense phase called the Planck epoch (all fundamental forces are unified – the period of Theory Of Everything (TOE)) at the end of which gravitational force separated from gauge forces, and passed through the Grand Unification Epoch and Inflationary Epoch (at the end of which strong forces separated from electroweak force) and the Electroweak Epoch (unification of electromagnetism and weak nuclear interaction). Other phases include – Baryogenesis, Hadron Epoch, Lepton Era, Nucleosynthesis, Photon Era, Recombination Epoch and presently Matter Dominated Era[7,14,23-25].

The generally acceptable mathematical theory for studying the evolution of the universe is general relativity[9]. General relativity is the theory of gravitation, in which gravitational effects between masses results from warping of space-time by the masses. In a uniform universe, general relativity has a simple solution for the evolution of the geometry of the universe (contraction or expansion) which depends on its content and past history[29]. In the presence of enough matter, the expansion will slow or even become a contraction. On the other hand, the dark energy (cosmological constant (Λ)) drives the universe towards increasing expansion[29]. The current rate of expansion is usually expressed as Hubble constant (H_0) with an estimated value of $H_0 = 100h\ kms^{-1}\ Mpc^{-1}$, with the Hubble parameter (h) having values of $h = 0.5 - 1.0$[24,25].

One of the fundamental goals of cosmology is to determine the expansion rate of the universe. Various methods that have been applied include:

(1) The use of distant Type 1a supernovae (SN 1a) as standard candles[22,2,8,31]. The apparent peak magnitude of these supernovae yields a relative luminosity distance d_L as a function of redshift from which the Hubble constant is estimated[27,28,32].

(2) Large galaxy surveys for mapping of cosmic distances and expansion, by using the large scale clustering pattern of galaxies which contains the signature of Baryon Acoustic Oscillation (BAOs). BAO refers to the regular periodic fluctuations in the density of the visible baryonic matter of the universe, caused by acoustic waves which existed in the early universe. By looking at large scale clustering of galaxies, a preferred length-scale which was imprinted in the distribution of photons and baryons propagated by the sound waves in the relativistic plasma of the early universe, can be calibrated by the observation of Cosmic Background Microwave Radiation (CMBR) and applied as cosmological standard rod (e.g.[1,5,19]).

(3) Other test include the redshift-angular size test, galaxy cluster gas mass fraction, strong gravitational lensing test and structure formation test, these test generally constrain cosmological parameters as a function of redshift (see review by Samushia & Rastra[26]).

Several decades have passed since Hubble published the correlation between distances to galaxies and their expansion velocities, but establishing an accurate cosmological distance scale and value for the Hubble constant (H_0) have proved challenging. The value of H_0 has evolved from $H_0 = 500\ kms^{-1}\ Mps^{-1}$ recorded by Hubble[17] to a well-known range of range of $H_0 = 50 - 100\ kms^{-1}\ Mps^{-1}$[3]. The Hubble Space Telescope (HST) gave a more specific value of $H_0 = 70 - 80 \pm 10\%$[11], while the WAMP data give $H_0 = 72 \pm 5\ kms^{-1}\ Mps^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$[30], where Ω_m is relative matter density (both luminous and dark matter) and Ω_Λ is the relative dark energy density.

In this article, we which to use the dependence of observed distance modulus ($m - M$) on redshift (z) and deceleration parameter (q_0) of the universe to constrain the Hubble constant for a large database obtained from NED and possibly trace the evolution of the Hubble parameter as a function of redshift ($H(z)$).

THEORETICAL RELATIONSHIPS

The evolutionary trend of the universe has been modeled by using the density parameter (Ω), determine by the density of the universe, the deceleration parameter (q_0) and the Hubble constant (H_0). The Hubble constant is an important parameter in cosmology as it not only determines its expansion rate, it also set limit to the possible age, critical density and size of the observable universe. The velocity (v) of the expansion of the universe has been defined by Hubble[16] as

$$v(t) = H_0(t)d \quad (1)$$

where d is the radius of the expanding universe. The size of the universe is unknown, yet it undergoes expansion or contraction, thus, the evolution of the universe can be express in terms of cosmic scale factor ($a(t)$) as

$$a(t) = \frac{R(t_e)}{R_0(t_0)} \quad (2)$$

In terms of the cosmic expansion factor (R_0), at the time (t_0), the Hubble constant is given by (e.g.[24])

$$H(t) = \frac{\dot{R}(t)}{R(t)} = \frac{\dot{a}(t)}{a(t)} \quad (3)$$

where $\dot{R}_0(t)$ is the first time derivative of R_0. The cosmic scale factor affect all distances, thus, the wavelength (λ_e) of a photon emitted at time of emission (t_e) and observed at another time (t_0), will be (e.g.[24])

$$\frac{\lambda_0}{\lambda_e} = \frac{R_0(t_0)}{R(t_e)} = [a(t)]^{-1} \quad (4)$$

where λ_0 is the wavelength of the observed photon at time (t_0). The cosmological redshift (z) is usually given by (e.g.[24])

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = [a(t)]^{-1} - 1 = H_0(t_0 - t_e) \quad (5)$$

where d is radius of the universe centered on the observer at time t_0, and c is the speed of photons. Allowing for some form of time evolution of the expansion factor ($R(t)$), we expand it using Taylor series. Following Roos[24] we can for t in general write

$$R(t) \approx R_0 + \dot{R}_0(t - t_0) + \frac{1}{2}\ddot{R}_0(t - t_0)^2 \quad (6)$$

Making use of the definition of equation (3), equation (6) implies that to a second order expansion, the cosmic scale factor can be written as

$$a(t) = 1 - \dot{a}_0(t_0 - t) + \frac{1}{2}\ddot{a}_0(t_0 - t)^2 =$$

$$1 - H_0(t_0 - t) + \frac{1}{2}\dot{H}_0(t_0 - t)^2 \quad (7)$$

From equation (3), we can write

$$\dot{R}_0(t_0 - t_e) = H_0 R_0(t_0 - t_e) \quad (8)$$

The deceleration parameter (q) which measure the rate of slowing down of the expansion factor is defined (e.g.[24]) by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} \quad (9)$$

Makin use of equations (3), (4), (7) and (9) in equation (5), the cosmological redshift can be expressed as

$$z(t) + 1 = \left[1 - \left\{H_0(t_0 - t_e) + \frac{1}{2}q_0 H_0^2(t_0 - t_e)^2\right\}\right]^{-1} \quad (10)$$

making use of the series expansion $(1-x)^{-1} \approx 1 + x + x^2$, equation (10) can be approximated to

$$z(t) = H_0(t_0 - t_e) + (1 + \frac{1}{2}q_0)H_0^2(t_0 - t_e)^2 \quad (11)$$

In obtaining equation (11), we made use of terms only to the second order in t. Inverting equation (11), making use of equation (3), we have H_0 in terms of redshift (z) as

$$H_0(t_0 - t_e) = z - \left(1 + \frac{1}{2}q_0\right)z^2 \quad (12)$$

For an isotropic and homogeneous universe, the Robertson–Walker metric in Minkowiski space-time best describe the geometry of space given by (e.g.[24])

$$ds^2 = c^2dt^2 - R(t)^2\left(\frac{d\sigma^2}{1 - k\sigma^2} + \sigma^2 d\theta^2 + \sigma^2 sin\,^2 d\varphi^2\right) \quad (13)$$

where ds is the invariant line element, σ is the dimensionless comoving coordinate, k is the curvature parameter, θ and φ are spherical coordinate points (polar and azimuthal angle respectively). In such homogenous isotropic universe, photons propagates along null geodesic given by $ds^2 = 0$, and along the line of sight of an observer, θ and φ are kept constant. Thus, the observed proper distance (d_p) to a galaxy for a flat universe with $k = 0$ is given by (e.g.[24])

$$d_p = c(t_0 - t_e)\left[1 + \frac{1}{2}H_0(t_0 - t_e)\right] \quad (14)$$

Using equation (12) in equation (14), the proper distance in terms of redshift (z) to the lowest order in ($t_0 - t_e$) is given by

$$d_p = \frac{cz}{H_0}\left[1 - \frac{1}{2}(1 + q_0)z\right] \quad (15)$$

The first term on the right of equation (15) gives the Hubble linear law, while the second term measures the deviation from linearity to lowest order depending on the value of q_0.

The luminosity distance (d_L) to a galaxy of absolute luminosity (L) with observed brightness (B_o) is given by (e.g.[24])

$$d_L = \sqrt{4\pi B_o L} \quad (16)$$

For an expanding universe parameterized by the cosmic scale factor ($a(t)$), in which photons are redshifted and suffer from energy effect, if the apparent brightness of a galaxy is B_a, then its proper distance is given by (e.g.[24])

$$d_p = (1 + z)\sqrt{4\pi B_a L} \quad (17)$$

Equating $B_a = B_o$, implies that $d_L = (1 + z)d_p$. Thus, the luminosity distance in terms of redshift is given by

$$d_L = \frac{cz}{H_0}\left[1 + \frac{1}{2}(1 - q_0)z\right] \quad (18)$$

In terms of distance modulus ($m - M$), the luminosity distance to a source is given by (e.g.[15])

$$m - M = 5\log\left(\frac{d_L}{10(pc)}\right) \quad (19)$$

Substituting equation (18) into equation (19), we have

$$m - M = 42.3852 - 5\log h + f(z) \quad (20)$$

where $f(z) = 5\log z + 5\log\left[1 + \frac{1}{2}(1 - q_0)z\right]$, and $H_0 = 100h\ kms^{-1}\ Mpc^{-1}$ (e.g.[24]) and in S.I is given by $H_0 = 3.241h \times 10^{-18}\ S^{-1}$. For $q_0 = 0$ (e.g.[18] – matter dominated universe)

$$f(z) = 5\log z + 5\log\left(1 + \frac{z}{2}\right) \quad (21a)$$

For $q_0 = -1$ (e.g.[18] – vacuum energy dominated universe)

$$f(z) = 5\log z + 5\log(1 + z) \quad (21b)$$

Equation (20) provides a way to constrain the Hubble parameter (h) for a given sample of sources with observed redshift and distance modulus.

ANALYSIS AND RESULTS

The data used in this analysis were sourced from the NASA Extragalactic Database (NED). We selected sources with observed distance modulus (m - M) and redshift (z). The sample consists of 11 585 sources which is spread over $0.00016 \leq z \leq 8.26$. Due to large volume of data, we binned the sources into various redshift bin widths to enable fair representation of sources in each bin.

The binning ranges, the mean, median values of each bin size and the estimated Hubble parameter and estimated age of the universe from Hubble parameter based on equation (21) are shown in Table 1. The plot of m - M against $f(z)$ for all the sources are shown in figures 1 and 2 for $q_0 = 0$ and $q_0 = 1$ respectively, the error bars are errors associated with the distance modulus. A linear fit to the plot gives: for $q_0 = 0$, we have $m - M = (1.02 \pm 0.07)f(z) + 43.23 \pm 0.25$, which gives $h = 0.93 \pm 0.1$; for $q_0 = -1$, we have $m - M = (0.95 \pm 0.07)f(z) + 42.76 \pm 0.20$, which gives $h = 0.75 \pm 0.07$. In general, using the minimum and the maximum values of the parameter in our data, the value of Hubble parameter we estimated lies in the range $0.6 \pm 0.2 \le h \le 2.2 \pm 0.7$, with an average value of $h = 0.67 \pm 0.22$ - 0.96 ± 0.29, giving an average age for the universe as 11.4 ± 2.4 - 14.3 ± 2.2 *Gyr*.

We also plotted the estimated Hubble parameter (h) against different redshift (z) bins to check the dependence of the Hubble parameter on different epoch (shown in figure 3). The plot showed an exponential dependence of h on z, with the best fit being $h = (0.71 \pm 0.36)e^{(0.21 \pm 0.09)z}$ and $h = (0.65 \pm 0.27)e^{(0.11 \pm 0.07)z}$ for $q_0 = -1$ and $q_0 = 0$ respectively.

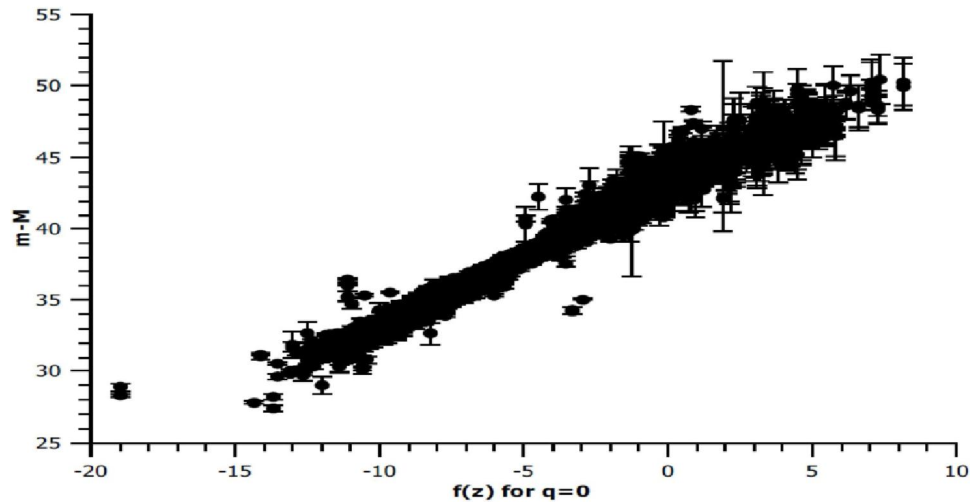

Figure 1 : Plot of m - M against $f(z)$ for $q_0 = 0$

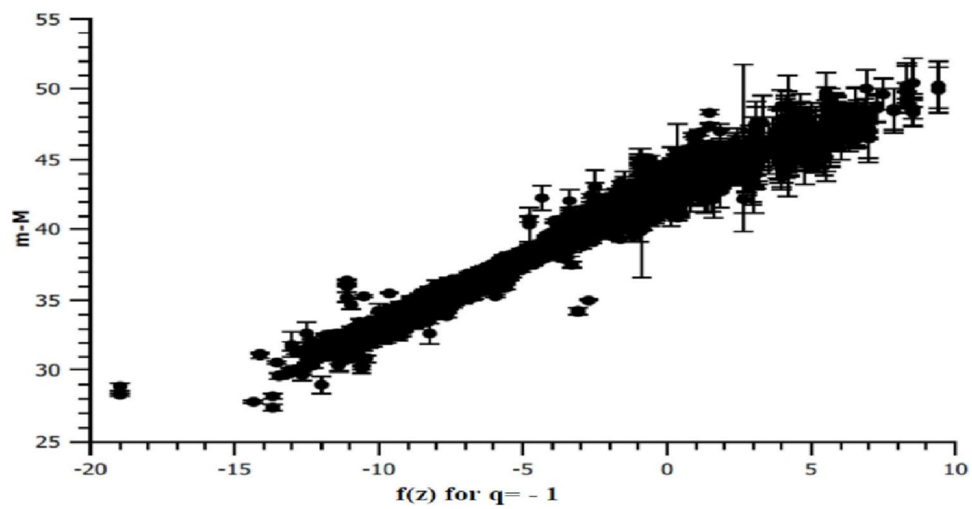

Figure 2 : Plot of m - M against $f(z)$ for $q_0 = -1$

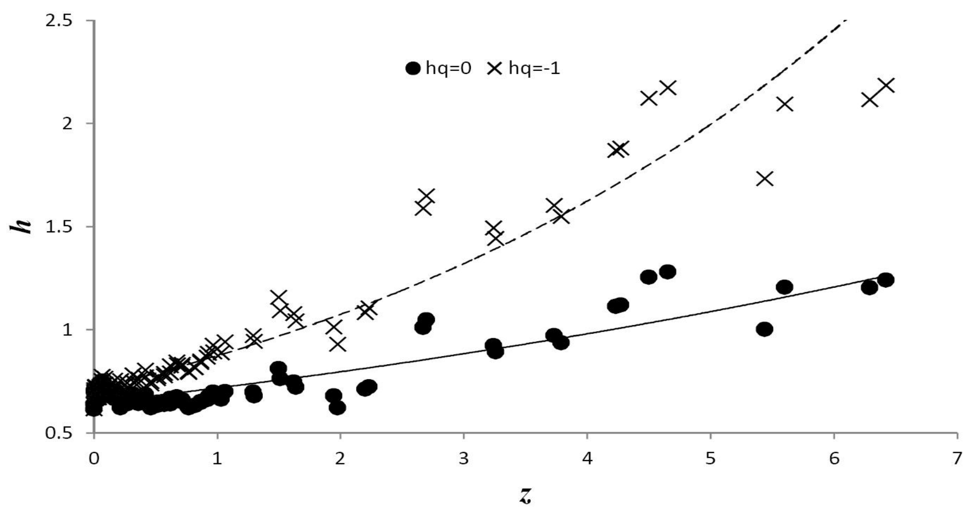

Figure 3 : The plot of estimated hubble parameter (h) against redshift (z)

DISCUSSION AND CONCLUSION

The value of Hubble constant (H_0) we estimated (i.e. converting the Hubble parameter (h) to Hubble constant) lies in the range of 60 ± 20 kms^{-1} $Mpc^{-1} \leq H_0 \leq 220 \pm 70$ kms^{-1} Mpc^{-1} with an average value of $H_0 = 67 \pm 22 - 96 \pm 29$ kms^{-1} Mpc^{-1}. The mean value of H_0 we estimated is in reasonable agreement with the HST value of $H_0 = 70 - 82 \pm 10\%$ kms^{-1} Mpc^{-1} recorded by Freedman[11]. Our result in general is also in agreement with results obtained from other works in literature e.g. $H_0 = 50 - 100$ kms^{-1} Mpc^{-1} by Assis et al.[3], $H_0 = 72 \pm 5$ kms^{-1} Mpc^{-1} from WMAP data by Spergel et al.[30], $H_0 = 74.1^{+7.9}_{-7.1}$ $kms^{-1}Mpc^{-1}$ by Aviles et al.[4], $H_0 = 79 \pm 30\%$ kms^{-1} Mpc^{-1} by Blinnikov et al.[6], $H_0 = 74 \pm 2.5$ kms^{-1} Mpc^{-1} by Lima et al.[18] similar to $H_0 = 74 \pm 2.5$ kms^{-1} Mpc^{-1} by Freedman et al.[12] and $H_0 = 68.9 \pm 7.1$ kms^{-1} Mpc^{-1} by Reid et al.[20]. The limit of the range 60 kms^{-1} $Mpc^{-1} \leq H_0 \leq 220$ kms^{-1} Mpc^{-1} is also in agreement with $H_0 = 60 - 220$ kms^{-1} Mpc^{-1} obtained by Farooq & Rastra[10].

Our plot of Hubble parameter (h) against redshift (z) indicates that h depends stronger on z for $q_0 = -1$ than for $q_0 = 0$, with an exponential dependence on redshift, an indication that the Hubble constant (H_0) is a function of time an indication that the earlier universe expanded faster (assuming redshift indicates the time evolution of the universe). The generally acceptable standard theory of the early universe – inflationary theory[13], incorporates an exponential increase in the very early evolution of the universe.

In conclusion, using the observed distance modulus and redshift, we estimated Hubble parameter, for different assumed energy densities of the universe represented by the deceleration parameter (q_0). The age of the universe estimated from the Hubble parameter suggests that the universe is $\sim 11.4 \pm 2.4 - 14.3 \pm 2.2$ Gyr old with the possible limit to the edge of the observable universe of $d \sim 5000$ Mpc.

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