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Competitive swim a short distance and optimization of energy conversion control module

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ABSTRACT

Under the premise of a range of reasonable conditions and assumptions, this study establishes athletes' energy transformation model and optimal control model for short-distance item in competitive swimming and conducts model parameters estimation and fitting combines with actual data to find the optimal solution. Then this article discusses the most optimal allocation of the athletes' physical strength and speed and points out that athlete should adopt the three-phase game strategy in competition. Specifically, the first phase is to accelerate with maximum thrust and reach the maximum speed at time; the second phase is maintaining uniform motion until at time, when ideally physical output is completed; the last phase is to decelerate and sprint. With this strategy the game can be completed in the shortest possible time, which provides theoretical support for technical training of the athletes and coaches.

KEYWORDS

Competitive swimming;
Strategy;
Optimal control;
Control model.

INTRODUCTION

With the increasing perfection of researches on swimming theories and techniques and scientificalness of training methods in the competitive swimming development at present, elite athletes from various countries generally increases their competitive level and the outcome of game often only lies between a fine line. Modern swimming is more than the competition of athletes' physical strength, speed and skills, but the competition result is largely related to the race strategy, the nature of which is a mathematical programming problem. The application of modern mathematics research methods in competitive sports began in the 1970s, when an American mathematician, T.B. Keller, built a mathematical model in 1973, for the training of middle and long distance athletes and achieves significant results. At the same time, Aiur combined the discus throw sport with the theory of mathematics, mechanics and computer science and improved the throwing techniques.

Optimal control theory is developed after World War II based on the concept of the state space. In the 1960s, with the rapid development of digital computer technology and space technology, optimal control theory began to take shape as an important branch of science, driven by the dynamic optimization theory. To the development today, it has made remarkable achievements in many fields, such as systems engineering, space technology and economic management. Optimal control theory is to maintain the target operated in accordance with its characteristic under particular admissible control conditions and to reach the optimal value for the target. The mathematical nature of this theory is a functional extremal problem, i.e. the variational problem under a set of constraints.

Modern competitive sport is not just a sport, but a game of comprehensive strength of physiology, psychology, mechanics and mathematics. Swimming, as a water exercise, is a low energy conversion efficiency item affected by many factors such as physical distribution, speed distribution and propulsion optimization, with only about 10% of the athlete's stigma converted to the forward thrust. Therefore, reasonable allocation of physical and impetus becomes a critical issue. In this study, how to maximize athlete's physical strength play and achieve good grades in competitive swimming is analyzed on the basis of optimal control theory, aiming at to provide new theoretical support for the optimization of athletic performance and promote swimming skills to a new level.

ENERGY CONVERSION MODEL IN SWIMMING

Competitive swimming is an important item in modern sports, which also represents a country's comprehensive strength of sports project. There are mainly two research directions in order to improve the athletic performance: one is the technical level, including increasing technology content, improving training methods and swimming technical movement and excavating the potential of athletes; body; the other is the tactical level, i.e. to reasonably use a variety of strategies and maximize their advantages into full play avoiding their weakness. This paper, mainly based on the tactical level, combines the optimal control theory with swimming and discusses how to play a fixed distance in the shortest possible time by rational allocation of limited physical strength. Establish a mathematical model and obtain the optimal solution, and the mathematical model consists of two parts:

1) Kinetic model

Swimming can be approximately seen as the movement only in the horizontal direction, and according to Newton's laws of motion:

$$mV(t) = F(t) - \frac{1}{2}A\rho V^2(t) \quad (1)$$

Wherein, $F(t)$ means thrust in horizontal direction, m is the athlete's mass, C_d is the drag coefficient, A stands for the projected area for the athlete's body, ρ is the density of water, V is the forward speed and the maximum value of it is hypothesized as Q, then $F(t)$ can be normalized as $u(t)$:

$$u(t) = \frac{F(t)}{Q} \quad 0 \le u(t) \le 1 \quad (2)$$

Through the regression analysis of A, Q and m, it is found that generally there is a strong relationship among them. And so indicate that:

$$a = \frac{\rho C_d A}{2m} \quad b = \frac{Q}{m}$$

The above two equations are substituted into formula (1), and the kinetic model can be simplified as:

$$V(t) = -aV^2(t) + bu(t) \quad (3)$$

2) Energy conversion model

Competitive swimming can be seen as an item of how to allocate the human energy $E(t)$ at the most reasonable way in a fixed distance. The changing of physical strength can be described as:

$$E(t) = K - P - N - R \quad (4)$$

Wherein K is the energy generated power, P is the propulsion power at the horizontal direction, N is the non-propulsion power, i.e. the consumed power by the resistance of water, R is wasting power in the form of heat. As the energy generated from anaerobic metabolism occupies the vast majority of the total energy in short-distance swimming competitions, it is assumed in this study that the energy generation in the match is totally from anaerobic metabolism and non-propulsion power and heat consumption are both ignored[6]. Besides, this study supposes that all energy consumed is on the propulsion at the horizontal direction. As a result, formula (4) can be simplified as:

$$E(t) = -b \cdot u(t) \cdot V(t) \quad (5)$$

ESTABLISHMENT OF OPTIMAL CONTROL MODEL

The pursued goal of swimming competitions is to swim a prescribed distance in the shortest possible time. For the convenience of further research, it is equally

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regarded as an issue of how to rationally allocate the physical strength and speed in the case of fixes time and energy to achieve the farthest swimming distance[9]. Assuming that athletes have the same starting and turning-back techniques and all set out in backstroke mode, i.e. the initial velocity is zero, then the optimal model for this problem is :

Steady nonlinear system equation of state $X = f$, namely:

$$X = \begin{bmatrix} V \\ E \end{bmatrix} = f = \begin{bmatrix} -aV^2(t) + bu(t) \\ -b \cdot u(t) \cdot V(t) \end{bmatrix} \quad (6)$$

Determination of propulsion $u(t)$: $0 \le u(t) \le 1$, when the system starts from the original state:

$$X(t)\big|_{t=0} = X_0 = \begin{bmatrix} 0 \\ E_0 \end{bmatrix}$$

And finishes at time t_f, and $E(t_f) \ge 0$

And with performance $J[u(\cdot)] = \int_0^{t_f} -Vdt$ indexreaches the minimum value.

MODEL SOLUTION

Judging from the nonlinearity of function at the right hand of the state equation as formula (6), the general method of determining the optimal solution of such function does not exist currently[2]. But there must be an optimal solution for swimming competition from the practical point of view. Based on Lagrange's theorem, the above performance indexes can be translated into:

$$J[u(\cdot)] = -\beta E(t_f) + \int_0^{t_f} \left[-V + \bar{\lambda}^T(t)(f - X)\right] dt \quad (7)$$

In formula (7): $\bar{\lambda}(t) = [\lambda_1(t), \lambda_2(t)]^T$ is association state vector, β is a non-negative constant and $\beta E(t_f) = 0$.

Construct Hamilton function:

$$H(X, \bar{\lambda}, u) = -V + \bar{\lambda}^T \cdot f = -V + \lambda_1(-aV^2 + bu) + \lambda_2(-buV) \quad (8)$$

Suppose $S(X(t_f)) = -\beta E(t_f)$, substitute it into formula (7), then:

$$J[u(\cdot)] = S(X(t_f)) + \int_0^{t_f} \left(H(X, \bar{\lambda}, u) - \bar{\lambda} X\right) dt \quad (9)$$

According to the ПонтрЯгин minimum principle, the association state equation is:

$$\begin{cases} \dot{\lambda}_1 = -\dfrac{\partial H}{\partial V} = 1 + 2a\lambda_1 V + \lambda_2 bu \\ \dot{\lambda}_2 = -\dfrac{\partial H}{\partial E} = 0 \end{cases} \quad (10)$$

Transversality conditions:

$$\begin{cases} \lambda_1(t_f) = -\dfrac{\partial S(X(t_f))}{\partial V(t_f)} = 0 \\ \lambda_2(t_f) = -\dfrac{\partial S(X(t_f))}{\partial E(t_f)} = -\beta \end{cases}$$

When $u(t) = u^*(t)$, H reaches the minimum value:

$$H\left(X^*(t), \bar{\lambda}(t), u^*(t)\right) = \min_{0 \le u(t) \le 1} H\left(X^*(t), \bar{\lambda}(t), u(t)\right) \quad (11)$$

And H is a constant on the optimal curve, namely:

$$H\left(X^*(t), \bar{\lambda}(t), u^*(t)\right) = H\left(X^*(t_f), \bar{\lambda}(t), u^*(t)\right) = const \quad (12)$$

As $t(f)$ is given, $\lambda_2(t) = -\beta$, $\dot{\lambda}_1 = 1 + 2a\lambda_1 V - \beta bu$, then

$$H(X, \bar{\lambda}, u) = -V - a\lambda_1 V^2 + (\lambda_1 + \beta V) bu \quad (13)$$

When other conditions remain unchanged, $u^*(t)$ is the optimal solution of H:

$$u^*(t) = \begin{cases} 1, \lambda_1 + \beta V < 0 \\ 0, \lambda_1 + \beta V > 0 \\ X, \lambda_1 + \beta V = 0 \end{cases} \quad (14)$$

When $\lambda_1 + \beta V = 0$, the optimal solution cannot be determined only by the above conditions. In this case, the optimal solution may be arbitrary value under $0 \le u^*(t) \le 1$.

When the time interval, $(t_i, t_j) \subset [0, t_f]$, length is non-zero, and it satisfies $\lambda_1 + \beta V = 0$, this optimization problem is singular; otherwise normal. When it is singular, the derivation of $\lambda_1 + \beta V = 0$ can be:

$$1 - 3a\beta V^2 = 0 \quad t \in (t_i, t_j) \quad (15)$$

When $\beta > 0$,

$$V(t) = \sqrt{\frac{1}{3a\beta}} \quad t \in (t_i, t_j) \quad (16)$$

$$u(t) = \frac{1}{3b\beta} \quad t \in (t_i, t_j) \quad (17)$$

It can be seen that when $\beta > 0$, singular situation may appear; when $\beta = 0$, generally normal circumstances occurs. The following is a discussion of different β situations.

1) When $\beta = 0$, the optimal solution is $u^* = \{1\}$, indicating that athlete should swim in the maximum propulsion and the speed is:

$$V(t) = \frac{\exp(rt) - 1}{\exp(rt) + 1} V_e \quad t \in [0, t_f] \quad (18)$$

Wherein $r = 2\sqrt{ab}$, $V_e = \sqrt{b/a}$, and the energy at finishing moment is:

$$E_0 + \int_0^{t_f} -bV_e \frac{\exp(rt) - 1}{\exp(rt) + 1} dt = E_0 - b\int_0^{t_f} V_e \frac{\exp(rt) - 1}{\exp(rt) + 1} dt > 0 \quad (19)$$

The result of the integral is the maximum forward distance at a given finishing time t_f. Full speed ahead with a speed of $u^* = \{1\}$ for arbitrary initial energy E_0 and moment when all energy is completely consumed is recorded as T_c. if $t_f \le T_c$, obviously $\{1\}$ is the optimal solution; but if $t_f > T_c$, it shows there is still energy unconsumed at last moment t_f and obviouslyis $\{1\}$ no longer the optimal solution, indicating that it is able to advance further away.

2) When $\beta > 0$, both normal solutions and singular solutions exist. It can be determined that normal solutions are: $\{1,0\}$, $\{1,0,1,0\}$, $\{1,0,1,0,1,0\}$ …, and singular solutions are: $\{1,u_c,0\}$, $\{1,u_c,1,0\}$, … wherein $u_c = 1/3b\beta$.

As can be seen from the above analysis, normal feasible solution and singular feasible solution are neither unique and it is very difficult to determine the optimal solution besides that E_0, t_f, a, b are all unknown parameters. The forward propulsion of swimming athletes in swimming process should not be frequently changing according to actual situations. Therefore, this article selects only the first few of the normal and singular solutions as feasible solutions and works out the relatively optimal solution among them.

MODEL PARAMETERS AND DETERMINATION OF OPTIMAL SOLUTION

The parameters of this model is calculated based on the 50M, 100M and 200M freestyle champion race results on the 27th Olympic Games, and $a = 0.32(m^{-1})$, $b = 1.79(N/Kg)$. On this basis, conduct parameter fitting process of the important parameter E_0. Supposing that the championship achievements of 50M, 100M and 200M are $T_{50} = 21.98s$, $T_{100} = 48.30s$ and $T_{200} = 105.35s$ respectively, the optimal solution when $t_f = T_{50}$ is $\{1\}$ and substitute $T_{50} = 21.98s$ into the following formula:

$$\int_0^{T_{50}} \frac{\exp(rt)-1}{\exp(rt)+1} V_e dt = 50 \quad \textbf{(20)}$$

It can be acquired that:

$V_e = \sqrt{b/a} = 2.37(m/s)$ $\quad b = aV_e^2 = 1.79(N/Kg)$ $\quad r = 2\sqrt{ab} = 1.52(1/s)$

$E_0 > 89.9(J/Kg)$

Similarly, when $T_{100} = 48.30s$, then there is $E_0 < 201.9(J/Kg)$, and it can be calculated that $E_0^* = 176(J/Kg)$ by fitting in least square method.

With the above parameters, the maximum distances corresponding to $t_f = 48.30s$ and $t_f = 105.35s$ are 107.55M and 202.01M respectively. Theoretically speaking, it shows that 100M and 200M champion athletes should achieve better results and there is still room for growth.

Using that above result of E_0^*, it can be determined that critical moment is $T_c = 42.23s$ and corresponding maximum distance is $D_{max} = 98.32$ M. When $t_f > T_c = 42.23s$, the optimal solution is $\{1,u_c,0\}$. As 100M, for example, when $t_f = 48.30s$, $\{1,u_c,0\}$ is the optimal solution and the corresponding propulsion function is:

$$u^* = \begin{cases} 1, t \in [0, t_1^-] \\ u_c, t \in [t_1^+, t_2^-] \\ 0, t \in [t_2^+, t_f] \end{cases} \quad \textbf{(21)}$$

Velocity distribution function is:

$$V^* = \begin{cases} \dfrac{2.37\exp(1.52t)-2.37}{\exp(1.52t)+1}, t \in [0, t_1^-] \\ V_c, t \in [t_1^+, t_2^-] \\ \dfrac{V_c}{0.32V_c(t-t_2)+1}, t \in [t_2^+, t_f] \end{cases} \quad \textbf{(22)}$$

In formula (22): $V_c = \dfrac{2.37\exp(1.52t_1)-2.37}{\exp(1.52t_1)+1}$ and $u_c = 0.18V_c^2$.

The optimal velocity curve is shown in Figure 1.

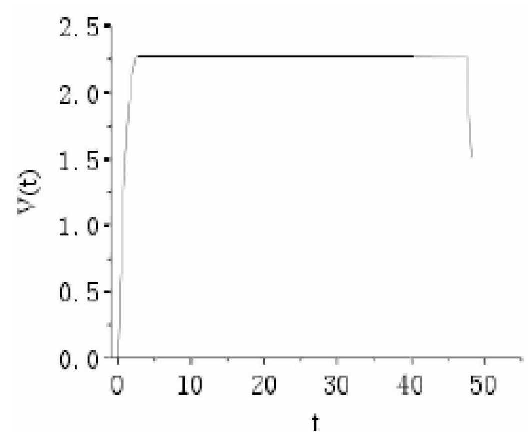

Figure 1 : Optimal velocity curve when $t_f = 48.30s$

It can be seen from the optimal solution $\{1,u_c,0\}$ that the entire time interval can be divided into three phases: in the first phase, accelerate with the greatest impetus

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and velocity changes from 0 to the maximum with a time period of 2.54s; the second phase is uniform motion with fixed thrust, the standardized thrust is 0.92 and the time reaching the optimal segmentation is 47.61s when the athlete's physical energy is completely consumed; the third phase is the decelerated coasting phase until the end, i.e. sprint stage.

The performance of athletes in the actual game is basically in line with this process. Just from the point of view of human control, it is difficult to achieve optimal control of thrust and thus accurately grasp the true optimal moment. Besides, since parameter fitting of E_0 in this article only refers to three sets of performance data as 50M, 100M, and 200M, the result is with certain limitations. Therefore, further research should take a larger sample data to do parameter fitting. Thus the more accurate the fitting result of E_0 is, the obtained optimal value will be much closer to the actual situation.

CONCLUSIONS

This article builds athletes' energy transformation model and optimal control model of short-distance item in competitive swimming and explores the optimal allocation plan of athletes' physical strength and speed by means of parameter estimation and fitting in the model. It is suggested that athletes should accelerate with a maximum thrust after starting off and reach the maximum velocity at time. Then maintain uniform motion after that moment until time, when physical energy is completely consumed in the ideal situations. Afterwards, the last phase is decelerated sprint, meaning to finish the race in the shortest possible time. In addition, as this article aims at short-distance item, due to the increase of the proportion of aerobic metabolism for long-distance item, just with appropriate adjustments of the model parameters, this model can still be used. In this study, combination of swimming and optimization theory provides new ideas for the study on swimming strategy and the conclusions from this research has great significance for swimming training and tactical arrangements.

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