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Biomechanical optimization model-based basketball field-goal percentage influence factors study

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ABSTRACT

Mathematical model is a key bridge that links mathematics with practice, for a complex practical problem, if it can find out a proper mathematical model then can simpler and clearly solve the complex problem, which also reflects strong applicability of establishing mathematical model. The paper establishes mathematical model on shooting problem by studying release angle, speed and other influence factors when basketball players shoot, finally it gets their impacts on field-goal percentage through analysis.

KEYWORDS

Basketball shooting; Release angle; Release speed; Hit rate; Mathematical model.

INTRODUCTION

By far, foreign research institutions analyses of basketball problems mainly start from combination, techniques, tricks, nutrition, physical exercises and other aspects, carry on training and cultivation from multiple aspects, and build groups of NBA level and world level basketball players, but to Chinese domestic basketball players, their height, physique and others multiple aspects keep paces with that of foreign players, therefore Chinese basketball athletes' field-goal percentage consideration factors should not be fully the same as that of foreign players analysis consideration factors. However, domestic works about such aspect researches are quite little, famous host Zhang Wei-Ping in "Hoop Park" made good illustration on Chinese players cultivation and hit rate improvements, but he didn't provide concretely about shooting movements and angles; other researchers mostly analyzed Chinese players from their attack and defense ability, hit rate, organizational ability and other aspects, their mainly analyzed population was CBA professional players and so lacked of general application. Therefore in such background, the paper combines with mechanical knowledge, utilizes mathematical model to discuss release speed, release angle, athletes and hoop distance, release height, basketball hoop entering angles when basketball player shoot, analyzes each factor impacts on field-goal percentage, and combines with ideal model, analyzes and sorts a conclusion that can simple shoot and also improve hit rate.

PROBLEM ANALYSES

It is well known that basketball running trajectory after shooting can be approximately regarded as a parabola, researches show if basketball running trajectory parabola is too high then basketball air flying time will be longer, air resistance and wind force impacts would be larger, so it is not easy to make the hoop; on the contrary, if basketball flying trajectory parabola is too low, then basketball angles that enter into the hoop will be smaller, and then it will also reduce field goal percentage. Therefore if field-goal percentage is related to basketball flying trajectory parabola height, main factors that decide basketball flying trajectory parabola height is release angle and release basketball speed. Therefore, the paper assume that ignore basketball shooters moods impacts on shooting result, and basketball shooters can correctly judge horizontal distance between shooting site and hoop, and make proper exertion; ignore air resistance, wind force impacts on flying trajectory after basketball releasing; assume that after shooting, basketball and hoop collision has no energy loss; assume that hoop and shooting parabola are in the same plane; ideally regard basketball as a particle.

MODEL ESTABLISHMENTS

In real life, it tends to ignore basketball and hoop sizes such simple cases, in mechanics, it regards basketball as particle (sphere center) oblique projectile movement. The paper supposes coordinates origin as sphere center P, lists out x (horizontal) direction and y (vertical)direction kinematic equations, and then it can get sphere center movement trajectory. So conditions that sphere center hits hoop center can be expressed as release speed, release height and release angle the three relationships, as well as the relationships between release angle and hoop incident angle, therefore we can get hoop incident angles and release angles according to different release heights and release speeds.

Because we ignore basketball and hoop size as well as air resistance impacts, the paper assumes basketball (when it doesn't release) sphere center pas coordinates origin, horizontal direction as x axis, vertical direction as y axis, at the time t=0 basketball player release speed is v! α is release angle, then basketball makes oblique projectile movements, and its kinematic equation is as following :

$$x(t) = v\cos\alpha t, \quad y(t) = v\sin\alpha t - \frac{gt^2}{2} \tag{1}$$

Among them, gravity accelerated speed is g ,so we get basketball sphere center movement trajectory (that is parabola):

$$y = x\tan\alpha - x^2\frac{g}{2v^2\cos^2\alpha} \tag{2}$$

Input x=L,y=H-h into formula (2), then it gets conditions that sphere center hits hoop center:

$$\tan\alpha = \frac{v^2}{gL}\left[1 \pm \sqrt{1 - \frac{2g}{v^2}\left(H - h + \frac{gL^2}{2v^2}\right)}\right] \tag{3}$$

The paper gets that after given release height h and release speed v ,then it will have two release angles α to meet mentioned conditions. Therefore formula (3) will change on the premise of having solutions:

$$1 - \frac{2g}{v^2}\left(H - h + \frac{gL^2}{2v^2}\right) \geq 0 \tag{4}$$

It can solve v and get:

$$v^2 \geq g\left[H - h + \sqrt{L^2 + (H-h)^2}\right] \tag{5}$$

Therefore, when height h assigns a certain numeric value, numeric value that meets formula (5) is minimum release speed $v_{\min}$, and it still is decreasing function about h .

By calculating formula (3),obtained two angles are respectively α_1 ! α_2 , and meet relationship $\alpha_1 > \alpha_2$. Therefore the paper gets that α_1 is binary increasing function about h and v .
When ball enters into hoop, incident angle β can be obtained from following formula:

$$\tan\beta = -\frac{dy}{dx}\bigg|_{x=L} \tag{6}$$

Derivative here inputs into formula (2) to calculate and can get:

$$\tan\beta = \tan\alpha - \frac{2(H-h)}{L} \tag{7}$$

So correspond to α_1 ! α_2 ,it has β_1 ! β_2 ,then $\beta_1 > \beta_2$.

When considering basketball and hoop sizes, as Figure 1, basketball diameter is d, hoop diameter is D.

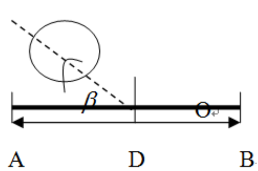

Figure 1 : Analysis of basketball entering into hoop

Obviously, even basketball movement trajectory is entering into hoop, but in case incident angle β is too small, basketball will collide hoop proximal A, it cannot make the hoop.

According to above figure, the paper gets two conditions that β should meet that sphere center hits hoop center and ball enters into hoop as:

$$\sin\beta > \frac{d}{D} \qquad (8)$$

Input d=24.6cm, D=45.0cm and get β >33.1 degree.

In previous calculation result, the cases that cannot meet the condition surely should be removed.

When basketball enters into hoop, sphere center can generate deflection from sphere center, deviation maximum distance is Δx in Figure, it can be calculated by incident angle β. By analyzing Δx ! x and α the three connections, the paper gets that release angle α accepted maximum deviation $\Delta\alpha$. Release speed v allowable maximum deviation Δv can be similarly processed.

By following Figure 2, it is clear that when basketball enters into hoop, maximum distance Δx that sphere center can deflect from front is:

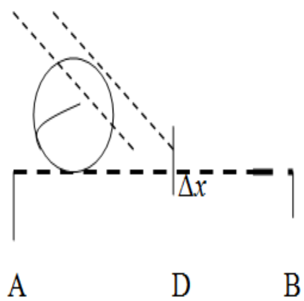

Figure 2 : Analysis of entering into hoop situation

$$\Delta x = \frac{D}{2} - \frac{d}{2\sin\beta} \qquad (9)$$

Among them deflect from back is similar to that of deflecting from the front.

In order to obtain release angle accepted maximum deviation $\Delta\alpha$, in formula (3) the paper uses $L \pm \Delta x$ to replace L to recalculate, but considering Δx includes β, so it will also include α, therefore the method cannot solve $\Delta\alpha$.

If start from formula (2) and input y=H-h, it can get:

$$x^2 \frac{g}{2v^2\cos^2\alpha} - x\tan\alpha + H - h = 0 \qquad (10)$$

Solve derivative of α and let x=L, it will have:

$$\left.\frac{dx}{d\alpha}\right|_{x=L} = \frac{L(v^2 - gL\tan\alpha)}{gL - v^2\sin\alpha\cos\alpha} \qquad (11)$$

The paper uses $\frac{\Delta x}{\Delta\alpha}$ to approximately replace above left derivatives, then it will get connection between release angle deviation $\Delta\alpha$ and Δx as following:

$$\Delta\alpha=\frac{gL-v^2\sin\alpha\cos\alpha}{L(v^2-gL\tan\alpha)}\Delta x \quad (12)$$

By $\Delta\alpha$ and obtained α, it also easily calculates relative deviation $\left|\frac{\Delta\alpha}{\alpha}\right|$.

Similarly, the paper uses formula (10) to solve derivative of v and meanwhile let $x=L$, then it will get release speed accepted maximum deviation:

$$\Delta v=\frac{gL-v^2\sin\alpha\cos\alpha}{gL^2}v\Delta x \quad (13)$$

By formula (12), (13), it gets v relative deviation as:

$$\left|\frac{\Delta v}{v}\right|=\left|\Delta\alpha\left(\frac{v^2}{gL}-\tan\alpha\right)\right| \quad (14)$$

RESULT ANALYSES

Different release heights corresponding release angles and minimum release speed

By above analysis, it is clear that formula (5) satisfied v is minimum release speed $v_{\min}$,then can calculate formula (3) and get corresponding release angle α_0 as:

$$\tan\alpha_0=\frac{v^2}{gL} \quad (15)$$

Take release height h=1.8~2.1(m), calculation result can refer to following TABLE 1.

TABLE 1 : Different release heights corresponding release angles and minimum release speeds

H (m)	v_{min} (m/s)	α_0 (degree)
1.8	7.6789	52.6012
1.9	7.5985	52.0181
2.0	7.5186	51.4290
2.1	7.4392	50.8344

Therefore the paper gets that correspond to minimum release angle and minimum release speed, the two will reduce followed by release height increases. Therefore, in general, release speed should not be lower than 8m/s.

Release angles and incident angles correspond to different release speeds and release heights

For release speed v=8.0~9.0(m/s) and release height 1.8~2.1(m),calculate by formula (3) release angles α_1 ! α_2, calculate by formula (7) incident angles β_1 ! β_2, result can refer to following TABLE 2.

TABLE 2 : Release angles and incident angles correspond to different release speeds and release heights

V (m/s)	H (m)	α_1	α_2	β_1	β_2
8.0	1.8	62.4099	42.7925	53.8763	20.9213
	1.9	63.1174	40.9188	55.8206	20.1431
	2.0	63.7281	39.1300	57.4941	19.6478
	2.1	64.2670	37.4019	58.9615	19.3698
8.5	1.8	67.6975	37.5049	62.1726	12.6250
	1.9	68.0288	36.0075	63.1884	12.7753
	2.0	68.3367	34.5214	64.1179	13.0240
	2.1	68.6244	33.0444	64.9729	13.3583
9.0	1.8	71.0697	34.1327	67.1426	7.6550
	1.9	71.2749	32.7614	67.7974	8.1663
	2.0	71.4700	31.3881	68.4098	8.7321
	2.1	71.6561	30.0127	68.9840	9.3472

According to above analysis, when β is above 33.1°basketball can be ensured to enter into the hoop. By above data, it is clear that all β_2 is smaller than33.1°,so it doesn't meet formula (8) precondition. Therefore, in case consider basketball and hoop size practical situation, release angle must be α_1.

It is well know, when speed is certain, release angle gets bigger, and then release angle will be bigger. However with speed increases, release height impacts on angles will be smaller, and generated impacts will be around 1 degree. When release height is certain, release speed gets bigger, then release angle will also be bigger, and generated impacts will be 7~9°.

Next step, the paper analyzes release speed and release angle the two maximum deviations. Combine with formula (12) and above obtained α_1, it can calculate release angle maximum deviations $\Delta\alpha$ and $\frac{\Delta\alpha}{\alpha}$, then combine with formula (13), (14),it gets release speed maximum deviations v and $\frac{\Delta v}{v}$,only lists h=1.8, 2.0(m) result into following TABLE 3.

TABLE 3 : Release angle and release speed maximum deviations

H (m)	α (degree)	V (m/s)	$\Delta\alpha$	Δv	$\left\|\frac{\Delta\alpha}{\alpha}\right\|$	$\left\|\frac{\Delta v}{v}\right\|$
1.8	62.4099	8.0	-0.7562	0.0528	0.0122	0.0065
	67.6975	8.5	-0.5603	0.0694	0.0082	0.0081
	71.0697	9.0	-0.4570	0.0803	0.0064	0.0089
2.0	63.7281	8.0	-0.7100	0.0601	0.0111	0.0075
	68.3367	8.5	-0.5411	0.0734	0.0079	0.0086
	71.4700	9.0	-0.4463	0.0832	0.0062	0.0092

To sum up, all allowable deviations are relative small. By further researching, it is clear that followed by speed gets bigger, angles allowable deviations would be smaller, speeds allowable deviations would be bigger, and it is strict with angles while loose with speeds. When release speed is certain, with heights being bigger, angles allowable deviations would be smaller, speeds allowable deviations would be bigger, it has lower requests on both speeds and angles.

MODEL EXPANSIONS

If paper adds air resistance impacts, then what change will release angles occurs to. By practical situation, it is clear that resistance will be very small, and then only need to proper simplification, similarly make each calculation as previous.

If we only add horizontal direction resistance, assume speed and resistance are in direct proportion, and proportion coefficient is k. The horizontal direction movement differential equation is as following:

$$\ddot{x} + k\dot{x} = 0, x(0) = 0, \dot{x}(0) = v\cos\alpha \tag{16}$$

Above equation solution is:

$$x(t) = v\cos\alpha \frac{1 - e^{-kt}}{k} \tag{17}$$

Considering practical resistance is very small (k will be no bigger than 0.05/ per second),time t will also very small (nearly 1 second),therefore the paper utilizes taylor formula to expand formula (17) (ignore vertical direction resistance, so y (t) is unchanged), it gets:

$$\begin{aligned} x(t) &= v\cos\alpha t - \frac{kv\cos\alpha t^2}{2} \\ y(t) &= v\sin\alpha t - \frac{gt^2}{2} \end{aligned} \tag{18}$$

When considering basketball and hoop sizes, condition that sphere center hits hoop center is decided by equation set:

$$\begin{cases} v\cos\alpha t - \dfrac{kv\cos\alpha t^2}{2} - L = 0 \\ v\sin\alpha t - \dfrac{gt^2}{2} - (H - h) = 0 \end{cases} \tag{19}$$

Similar to above model solution then it can solve different release speeds and release heights' release angles and incident angles, it will not further discuss here.

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