

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(9), 2014 [2965-2972]

Based on best lineup of LINGO optimization model of the gymnastics team research

Yu Peng*, Guoqing Zhang

Institute of Physical Education, Hunan Institute of Science and Technology, Yueyang 414006, Hunan, (CHINA)

ABSTRACT

The women's gymnastics team by the uneven bars, the balance beam, the vaulting horse and the floor exercise four. In the actual competition, the athlete's strength size, inevitably determines the performance of team scores, but a good lineup also have very important influence to the result of the match. Team problem in this paper, the women's gymnastics team competition carried on the thorough discussion, first of all, according to the known conditions are the most pessimistic situation of each player's score, to establish the objective function makes the total score as high as possible and get constraint conditions in combination with the athletes in the history of the rules, established by using integer programming model, the model is solved using the software, get pessimistic model best lineup; finally applies probability statistical theory and normal distribution knowledge to further deepen problem, by calculating, it gets variance D_{ij} and expectation $C_{i,j}$, and inputs them into standard normal programming functions so that can get objective functions, applies *lingo* software to solve best starting line-up.

KEYWORDS

Best line-up; programming; Probability and statistics; Normal distribution.



INTRODUCTION

Gymnastics World Cup is one of the TiCaoJie top events after the Olympic Games and world championships, added to the list of the international gymnastics federation official yearbook. Athens in 2004, in the case of the delegation overfulfilled task, with seven gold MEDALS for the Chinese gymnastics team became the worst team. After returning, the Chinese gymnastics team starting from negative, hard work, from the intravenous drip, small to each movement and life details, big to the adjustment of the coach team division of labor, as well as the analysis of each athlete battlefield, overall layout to adjust the lineup of the players. Until October 2006, in Aarhus, Denmark, from negative to start the Chinese gymnastics eight gold MEDALS at last shocked the world! Two years Chinese gymnast from negative to "shock" wins in the squad, the couple use strength to prove the Chinese gymnastics team is a good fight collective. It is not empty talk, but if no pay, no arrangement good squad, so the whole team play and even the title will be affected, therefore, to master each contestant's score data, discharge reasonable lineup is very important to team performance.

Yu-bao zhang, Cheng Zaikuan characteristics of the 29th Olympic Games men's competitive gymnastics competition tactics study (2009)^[1]. Ling-feng kong, Hu Haixu chang-liang du (2010) for the 2008 Olympic Games men's gymnastics team final before six grades in vertical and horizontal, total score, inside and outside the formation of examples analysis, summed up the similar international high level of men's gymnastics team final on comprehensive competitive performance reflect the characteristics of competition tactics to win^[2]. Zhang wei, Yuan Chi Ye Yanqing Jia Xiang, Dai Qiwei (2012) analyzed how the existing statistical data on the premise of a gymnastics team athletes squad problem^[3]. Wang Zhirui, tan Lin (2013) through the literature study of gymnastics team tactics, found the gymnastics tactics only used in the game, not only in the run-up to the training, to adapt to the training and the competition has the tactical use of^[4].

The paper goes into deeper exploration on women's gymnastics team forming problem, the whole paper overall adapts optimization thought and targeted at different problems, respectively applies integer programming model and probability statistical theory to establish corresponding model, combines with *lingo*, *matlab* and other software programming to solve, and gets best line-up arrangement way in different cases.

PESSIMISTIC MODEL ESTABLISHMENT AND SOLUTION

In probability theory, random variable X_1, X_2, \dots, X_n are pair wise independent, if $Y = X_1 + X_2 + \dots + X_n$, then:

Introduce expectation: $E(Y) = E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n)$

Introduce variance: $D(Y) = D(X_1 + X_2 + \dots + X_n) = D(X_1) + D(X_2) + \dots + D(X_n)$

Introduce normal distribution formula:
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard normal curve $N(0,1)$ is a kind of special normal distribution curve, and standard normal population value probability in any one interval (a,b). Standard normal distribution is a kind of special normal distribution, standard normal distributed μ and σ^2 are 0 and 1, it often uses ξ (or Z) to show variable conforms to standard normal distribution, it records as $z \sim N(0,1)$. In general, normal distribution and standard normal distribution transformation: due to general normal population its image is not surely symmetric to y axis, to any one normal population, the probability of its value less than x. Only need to use it to solve normal population probability in one specific interval.

Establish 0 ~ 1 variable model

By consulting literature, it gets athlete each item score and probability distribution table general status, as TABLE 1.

TABLE 1 : Athlete each item score and probability distribution table

Item	1	2	3	4	5
High-low bars	8.4~0.15	9.3~0.1	8.4~0.1	8.1~0.1	8.4~0.15
	9.2~0.25	9.5~0.1	8.8~0.2	9.1~0.5	9.5~0.5
	9.4~0.1	9.6~0.6	9.0~0.6	9.3~0.3	9.2~0.25
	9.5~0.5	9.8~0.2	10~0.1	9.5~0.1	9.4~0.1
Balance beam	8.4~0.1	8.4~0.15	8.1~0.1	8.7~0.1	9.0~0.1
	8.8~0.2	9.0~0.5	9.1~0.5	8.9~0.2	9.2~0.1
	9.0~0.6	9.2~0.25	9.3~0.3	9.1~0.6	9.4~0.6
	10~0.1	9.4~0.1	9.5~0.1	9.9~0.1	9.7~0.2
Vaulting horse	9.1~0.1	8.4~0.1	8.4~0.15	9.0~0.1	8.3~0.1
	9.3~0.1	8.8~0.2	9.5~0.5	9.4~0.1	8.7~0.1
	9.5~0.6	9.0~0.6	9.2~0.25	9.5~0.5	8.9~0.6
	9.8~0.2	10~0.1	9.4~0.1	9.7~0.3	9.3~0.2
Floor exercises	8.7~0.1	8.9~0.1	9.5~0.1	8.4~0.1	9.4~0.1
	8.9~0.2	9.1~0.1	9.7~0.1	8.8~0.2	9.6~0.1
	9.1~0.6	9.3~0.6	9.8~0.6	9.0~0.6	9.7~0.6
	9.9~0.1	9.6~0.2	10~0.2	10~0.1	9.9~0.2
Item	6	7	8	9	10
High-low bars	9.4~0.1	9.5~0.1	8.4~0.1	8.4~0.15	9.0~0.1
	9.6~0.1	9.7~0.1	8.8~0.2	9.5~0.5	9.2~0.1
	9.7~0.6	9.8~0.6	9.0~0.6	9.2~0.25	9.4~0.6
	9.9~0.2	10~0.2	10~0.1	9.4~0.1	9.7~0.2
Balance beam	8.7~0.1	8.4~0.1	8.8~0.05	8.4~0.1	8.1~0.1
	8.9~0.2	8.8~0.2	9.2~0.05	8.8~0.1	9.1~0.5
	9.1~0.6	9.0~0.6	9.8~0.5	9.2~0.6	9.3~0.3
	9.9~0.1	10~0.1	10~0.4	9.8~0.2	9.5~0.1
Vaulting horse	8.5~0.1	8.3~0.1	8.7~0.1	8.4~0.1	8.2~0.1
	8.7~0.1	8.7~0.1	8.9~0.2	8.8~0.2	9.2~0.5
	8.9~0.5	8.9~0.6	9.1~0.6	9.0~0.6	9.4~0.3
	9.1~0.3	9.3~0.2	9.9~0.1	10~0.1	9.6~0.1
Floor exercises	8.4~0.15	8.4~0.1	8.2~0.1	9.3~0.1	9.1~0.1
	9.5~0.5	8.8~0.1	9.3~0.5	9.5~0.1	9.3~0.1
	9.2~0.25	9.2~0.6	9.5~0.3	9.7~0.5	9.5~0.6
	9.4~0.1	9.8~0.2	9.8~0.1	9.9~0.3	9.8~0.2

When every player each single item score is estimated on the premise of best pessimistic status, solves best starting line-up, so actually it should solve on which line-up start the team total score can be highest. Firstly extract every athlete estimated performance in most pessimistic state, details refer to TABLE 2.

TABLE 2 : Members score performance estimation under most pessimistic state

Item	1	2	3	4	5	6	7	8	9	10
High-low bars	8.4	9.3	8.4	8.1	8.4	9.4	9.5	8.4	8.4	9.0
Balance beam	8.4	8.4	8.1	8.7	9.0	8.7	8.4	8.8	8.4	8.1
Vaulting horse	9.1	8.4	8.4	9.0	8.3	8.5	8.3	8.7	8.4	8.2
Floor exercises	8.7	8.9	9.5	8.4	9.4	8.4	8.4	8.2	9.3	9.1

According to above analysis of the problem, establish model as following:
 $i : 1, 2, \dots, 10$, represents ten athletes serial number;

$j:1,2,3,4$, successively represent sports items high-low bars, balance beam, vaulting horse and floor exercises.

In the model, set 0~1 variable x_{ij} and y_i to assist to establish model

$$x_{ij} = \begin{cases} 0 & \text{athlete } i \text{ do not attend the } j\text{th sport} \\ 1 & \text{athlete } i \text{ attends the } j\text{th sport} \end{cases}$$

In addition, in order to easy to solve model, the model needs to introduce the second 0~1 integer

variable y_i ,

$$y_i = \left[\frac{1}{4} \sum_{j=1}^4 x_{ij} \right]$$

In order to fix its value range in the range of 0~1, may as well do effective processing, after solving integers, it can be expressed as

$$y_i = \begin{cases} 0 & \text{athlete } i \text{ do not attend the all-around gymnastics} \\ 1 & \text{athlete } i \text{ attend the all-around gymnastics} \end{cases}$$

Every item can have 6 players to participate, the best case is full of people, then: $\sum_{j=1}^{10} x_{ij} = 6$

Every team has four people to participate in all-round competition, then: $\sum_{i=1}^{10} y_i = 4$

Objective function is highest team total score, list formula as following: $\max z = \sum_{i=1}^{10} \sum_{j=1}^4 a_{ij} x_{ij}$

Among them, a_{ij} represents the i athlete score that participates in the j item.

Set in case of pessimistic, the i athlete participates in the j item score is b_{ij} , then $a_{ij} = b_{ij}$. By above conditions, through above and analysis process, it is clear that under most pessimistic status, the paper established 0~1 integer programming model is as following:

$$\max z = \sum_{i=1}^{10} \sum_{j=1}^4 a_{ij} x_{ij}$$

$$s.t. \begin{cases} \sum_{i=1}^{10} x_{ij} = 6 \\ \sum_{i=1}^{10} y_i = 4 \\ (1 - y_i) \cdot \sum_{j=1}^4 x_{ij} \leq 3 \\ x_{ij} = 0,1 \end{cases}$$

Model solution

By *lingo* programming, it gets under most pessimistic status, the team starting line-up and each athlete score status can refer to TABLE 3.

TABLE 3 : Each team member starting and score status under most pessimistic status

Item	1	2	3	4	5	6	7	8	9	10
High-low bars		9.3			8.4	9.4	9.5		8.4	9.0
Balance beam		8.4		8.7	9.0	8.7		8.8	8.4	
Vaulting horse	9.1	8.4		9.0	8.3	8.5			8.4	
Floor exercises		8.9	9.5		9.4	8.4			9.3	9.1

According to above TABLE 3, it is easily known that under most pessimistic status, players that participate in all-round item are No. 2, 5, 6, 9 athletes, other players participate in single item competition. In this case, it can let each player to get highest total score in scoring most pessimistic state, as TABLE 4.

TABLE 4 : Best starting line-up

Item	First item player	Second item player	Third item player	Fourth item player	All-round item player
Player	7, 10	4, 8	1, 4	3, 10	2, 5, 6, 9

Model establishment

Firstly analyze if according to previous information and recent each kind of information, winning team total score is no less than 236.2 points this time, because if items are 24 pieces at most and all participate total score is just 240 points, it must participate in whole 24 items if it wants to win this time. To get starting line-up meet conditions and win, then it needs probability that team total score to be no less than 236.2 points to be largest, then the problem can be converted into solution:

$$\text{Max } P\left(\sum_{i=1}^{10} \sum_{j=1}^4 a_{ij} x_{ij} \geq 236.2\right) \quad (i = 1, \dots, 10, j = 1, 2, \dots, 4)$$

Among them a_{ij} represents the i athlete participates in j item's score, x_{ij} represents whether the i athlete participates in the j item or not

Set team total score is:
$$S = \sum_{i=1}^{10} \sum_{j=1}^4 a_{ij} x_{ij}$$

Then score expectation is:
$$E(S) = \sum_{i=1}^{10} \sum_{j=1}^4 c_{ij} x_{ij}$$

Among them, c_{ij} is the i athlete participating in j item event's score average value.

Score variance is :
$$D(S) = \sum_{i=1}^{10} \sum_{j=1}^4 D_{ij} x_{ij}$$

Among them, D_{ij} is the i athlete participating in the j item event score variance (refer to TABLE 5).

TABLE 5 : Participated athlete each item scoring variance

AthleteItem	1	2	3	4	5	6	7	8	9	10
High-low bars	0.1425	0.0180	0.1440	0.1280	0.1425	0.0180	0.0180	0.1440	0.1425	0.0380
Balance beam	0.1440	0.0800	0.1280	0.0880	0.0380	0.0880	0.1440	0.0840	0.1520	0.1280
Vaulting horse	0.0380	0.1440	0.1425	0.0380	0.0720	0.0320	0.0720	0.0880	0.1440	0.1280
Floor exercises	0.0880	0.0380	0.0180	0.1440	0.0180	0.1425	0.1520	0.1580	0.0320	0.0380

By converting problems, it finds the problem conforms to normal distribution, from which

$$\mu = E(S) \quad \sigma = \sqrt{D(S)}$$

$$u = \frac{S - E(S)}{\sqrt{D(S)}}$$

Convert non-standard normal distribution into standard normal distribution

$$u_0 = \frac{236.2 - \sum_{i=1}^{10} \sum_{j=1}^4 c_{ij} x_{ij}}{\sqrt{\sum_{i=1}^{10} \sum_{j=1}^4 D_{ij} x_{ij}}}$$

Among them,

Due to score probability conforms to normal distribution, probability that the team total score no less than 236.2 points is: $P(u \geq u_0) = 1 - \phi(u_0)$

To random variable Y that conforms to standard normal distribution, when $Y \leq x$, its score density function is increased single-valued function, so solve $1 - \phi(u_0)$ maximum value can be

converted into solving u_0 minimum value that objective function, $\frac{236.2 - \sum_{i=1}^{10} \sum_{j=1}^4 c_{ij} x_{ij}}{\sqrt{\sum_{i=1}^{10} \sum_{j=1}^4 D_{ij} x_{ij}}}$ minimum value. The

$$\min \frac{236.2 - \sum_{i=1}^{10} \sum_{j=1}^4 c_{ij} x_{ij}}{\sqrt{\sum_{i=1}^{10} \sum_{j=1}^4 D_{ij} x_{ij}}} \quad (i = 1, 2, \dots, 10, j = 1, 2, \dots, 4)$$

optimization model that meets conditions is :

$$s.t. \left\{ \begin{array}{l} \sum_{i=1}^{10} x_{ij} = 6 \\ \sum_{i=1}^{10} y_i = 4 \\ (1 - y_i) \sum_{j=1}^4 x_{ij} \leq 3 \\ x_{ij} = 0, 1 \end{array} \right.$$

STANDARD NORMAL DISTRIBUTION MODEL SOLUTIONS

The same as 0-1 model, use lingo to program, solve and get TABLE 6.

TABLE 6 : Winning starting line-up

Participated Item	Participated first item player	Participated second item player	Participated third item player	Participated fourth item player	Participated all-round item player
Participated member	6, 7	1, 8	1, 4	6, 8	3, 5, 9, 10

Then solve score expectation $E = 224.6$, $(u_0)_{\min} = 7.918$, $P(u \geq u_0) = 1 - \phi(u_0) \approx 0$, Because solved winning probability is nearly equal to 0, and the probability is arranged line-up maximum value that meets the conditions,so start with the line-up, winning probability is almost zero, and nearly cannot win. By Matlab,it draws score normal distribution Figure 1,it can verify results.

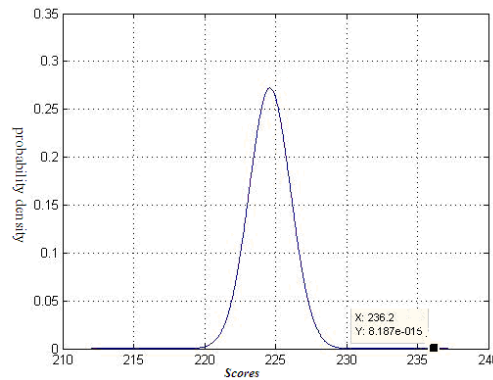


Figure 1 : Score normal distribution chart that meets conditions

If it wants to have 90% confident to combat opponents, then it should have $P(S > w) \geq 90\%$ hat

conforms to normal distribution, so it has :

$$1 - \phi\left(\frac{w - E(S)}{\sqrt{D(S)}}\right) \geq 90\%$$

Search normal distribution table and can get:

$$\frac{w - E(S)}{\sqrt{D(S)}} \leq -1.28$$

Through deformation simplifying, sort and get:

$$w \leq \sum_{i=1}^{10} \sum_{j=1}^4 c_{ij} x_{ij} - 1.28 \sqrt{\sum_{i=1}^{10} \sum_{j=1}^4 D_{ij} x_{ij}}$$

Only need to solve subsequent maximum value that can solve u value. Solved optimization model is :

$$\max \sum_{i=1}^{10} \sum_{j=1}^4 c_{ij} x_{ij} - 1.28 \sqrt{\sum_{i=1}^{10} \sum_{j=1}^4 D_{ij} x_{ij}} \quad (i = 1, \dots, 10, j = 1, 2, \dots, 4)$$

$$s.t. \begin{cases} \sum_{i=1}^{10} x_{ij} = 6 \\ \sum_{i=1}^{10} y_i = 4 \\ (1 - y_i) \cdot \sum_{j=1}^4 x_{ij} \leq 3 \\ x_{ij} = 0, 1 \end{cases}$$

Apply lingo into programming, solve maximum value as 223.3301, and further get $u \leq 223.3301$ Then best starting line-up is TABLE 7.

TABLE 7 : Best starting line-up

Item	First item player	Second item player	Third item player	Fourth item player	All-round item player
Player	2,7	5, 8	1, 4	2, 5	3,6, 9, 10

To sum up, it can get 90% confident to defeat player that is not bigger than 223.3301points. By Matlab,it draws scores normal distribution Figure 2, it can verify results.

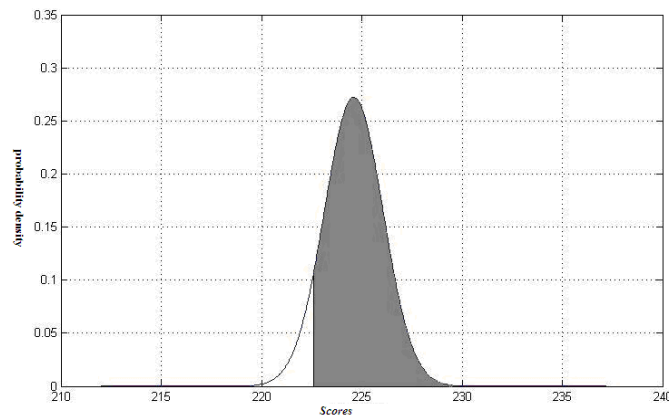


Figure 2 : Score normal distribution chart that meets conditions

CONCLUSION

This model considers each player scores and probability, gives a reasonable mathematical model of 0-1 integer programming. The model will be complicated squad selection problem, jump out from the combination of a large amount of data, using 0-1 integer programming, will be reduced to a simple optimization problems. Will at the same time, using the theory of probability estimation problem with combined optimization model, the calculation is simple, clear, easy to understand. Not only played a maximum value of each member, but also future prospects for the title and score at the reasonable estimates.

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