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## Baseball best hitting point model research based on angular momentum conservation and elastic beam model

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### ABSTRACT

The paper considers from baseball player available hitting maximum speed and baseball to hand minimum impact force when hitting (that generates minimum injury to athletes) these two aspects, it respectively establishes best hitting point models. Maximum hitting speed point is solved by momentum and angular momentum theorem as well as other typical theoretical mechanical models; bat to athlete minimum acting force is solved by material mechanics elastic beam model. By additional weight coefficient, it defines best hitting point model which can produce maximum hitting speed meanwhile it generates minimum acting force on athlete, and verifies model stability to weight coefficient. So that it denies people concept that best hitting point doesn't lie in bat top terminal, which provides scientific evidence for baseball players' better performance in game as well as more scientific and effective training. © 2014 Trade Science Inc. - INDIA

### KEYWORDS

Best hitting point;  
Momentum conservation;  
Angular momentum  
conservation;  
Elastic beam model.

### INTRODUCTION

Baseball is a collective and stronger confrontation ball sport event with main feature as bat playing ball; it is widely developing internationally with great influences, which is honored as "competitiveness and intelligence combination". Every baseball player wishes to get good result in game, there are many factors affect baseball performance, the maximum factor of which is hitting position. Based on mechanical torque theory, it can simply proves, "best hitting point" is in the terminal of bat larger end, but practice proves that best hitting point doesn't lie in the position.

By far most of researches on the problems get involved in best hitting point that can hit maximum speed, but meanwhile investigation shows baseball player sports injury rate has already arrived at as high as 56%, and

above 40% players' injury concentrates on hands and shoulder, which shows baseball player hitting motions generate certain degree injury on their body. Therefore, this paper will consider from baseball player available hitting maximum speed and baseball to hand minimum impact force when hitting (that generates minimum injury to athletes) these two aspects, it respectively establishes best hitting point models. And by additional weight coefficient, it defines best hitting point model which can produce maximum hitting speed meanwhile it generates minimum acting force on athlete, and verifies model stability to weight coefficient.

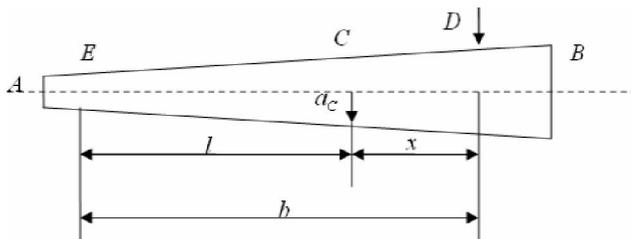
### BEST HITTING POINT MODEL

**Only consider maximum hitting speed best hitting**

**point model**

To get better results, batter should let ball get larger hitting speed<sup>[1]</sup>. Therefore, it firstly explores maximum speed corresponding best hitting point. By torque simple principle, it deduces that best hitting point is the top of club, in the following; it defines best hitting point by model.

To convenient for model calculating, we simplify model as Figure 1 showed circular table, from which, E is hand gripping bat position, C is bat mass center, D is bat and ball contact point<sup>[2]</sup>.



**Figure 1: Bat simplified model**

Due to hitting instantaneous, bat rotates around E point, angular speed is  $\omega$ , therefore mass center speed is  $u = \omega \times l$ , hitting point speed is  $v = \omega(l + x)$ , according to momentum conservation and angular momentum conservation as well as restitution coefficient expressions, it has<sup>[3]</sup>:

$$m_1 v_1 + m_2 u_1 = m_1 v_2 + m_2 u_2 \tag{1}$$

$$m_1 v_1 x + J \omega_1 = m_1 v_2 x + J \omega_2 \tag{2}$$

$$e = \frac{u_2 - v_2}{v_1 - u_1}, \text{ while because hitting point is not mass}$$

center, we improve restitution coefficient expression as:

$$e = \frac{u_2 - x\omega_2 - v_2}{v_1 - u_1 - x\omega_1} \tag{3}$$

By(1)(2)(3), it can get ball speed expression after hitting as following:

$$v_2 = v_1 + \frac{m_2 J (1 + e) u_1 - 2 m_2 J v_1 + m_2 J \omega_1 (1 + e) x}{(m_1 + m_2) J + m_1 m_2 x^2}$$

Information consulting parameters can refer to TABLE 1:

In the following, it needs to solve bat rotational inertia:

To solve bat mass center and rotational inertia, ex-

**TABLE 1: Baseball and bat parameters**

Parameter	$m_1$	$v_1$	$u_1$	$\omega_1$	$e$
Value	0.142kg	-40m/s	16m/s	34rad/s	0.4

tend above bat model to cone, and then bat volume is equal to big cone volume minus small cone volume.

$$\text{Among them: } h = \frac{R_2}{R_2 - R_1} h_2, h_1 = \frac{R_1}{R_2 - R_1} h_2$$

$$V = \frac{\pi}{3} R_2^2 h - \frac{\pi}{3} R_1^2 h_1 = \frac{\pi}{3} \frac{h_2}{R_2 - R_1} (R_2^3 - R_1^3) \tag{4}$$

Bat mass is:

$$m_2 = \rho V \tag{5}$$

In formula:  $\rho$  is bat density, V is bat volume.

Mass center coordinate formula is:

$$x_c = \frac{\int x dm}{m} = \frac{3}{4} \cdot \frac{R_2^4 - R_1^4}{R_2^3 - R_1^3} \cdot \frac{h_2}{R_2 - R_1} \tag{6}$$

$$l = x_c - \frac{R_1 h_2}{R_2 - R_1} \tag{7}$$

While because  $J = \int x^2 dm$  big cone inertia moment is:

$$J_{C2} = \frac{3}{80} \rho \left( \frac{\pi}{3} \frac{R_2^3 h_2}{R_2 - R_1} \right) (4R_2^2 + h_2^2) + \rho \left( \frac{\pi}{3} \frac{R_2^3 h_2}{R_2 - R_1} \right) \left( \frac{3}{4} \frac{R_2 h_2}{R_2 - R_1} - x_c \right)^2 \tag{8}$$

Small cone inertia moment is:

$$J_{C2} = \frac{3}{80} \rho \left( \frac{\pi}{3} \frac{R_2^3 h_2}{R_2 - R_1} \right) \left[ 4R_1^2 + \left( \frac{R_1 h_2}{R_2 - R_1} \right)^2 \right] + \rho \left( \frac{\pi}{3} \frac{R_2^3 h_2}{R_2 - R_1} \right) \left( \frac{3}{4} \frac{R_1 h_2}{R_2 - R_1} x_c \right)^2 \tag{9}$$

Mass center inertia moment is:

$$J_C = J_{C2} - J_{C1} \tag{10}$$

E point inertia moment is:

$$J_E = J_C + ml^2 \tag{11}$$

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Collecting information, it is clear that small radius  $R_1$  is 12.55mm, big radius  $R_2$  is 35mm, bat length  $h_2$  is 1050mm, wood bat density  $\rho$  is  $700kg/m^3$ . Input (1)~(11) into speed  $v_2$  following hitting point to mass center distance  $x$  changing figure as Figure 3.

From Figure 3, it is clear that ball hitting speed firstly increases and then decreases with distance  $x$  changes, it is obvious that best hitting point is not in the top of club, when  $x=0.074m$ , speed arrives at maximum that is 54.57m/s.

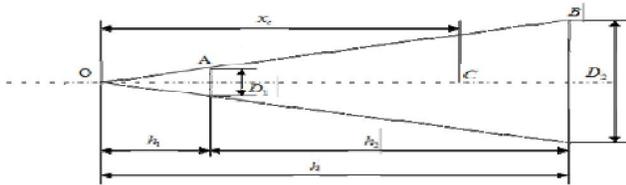


Figure 2 : Calculate bat inertia moment simplified model

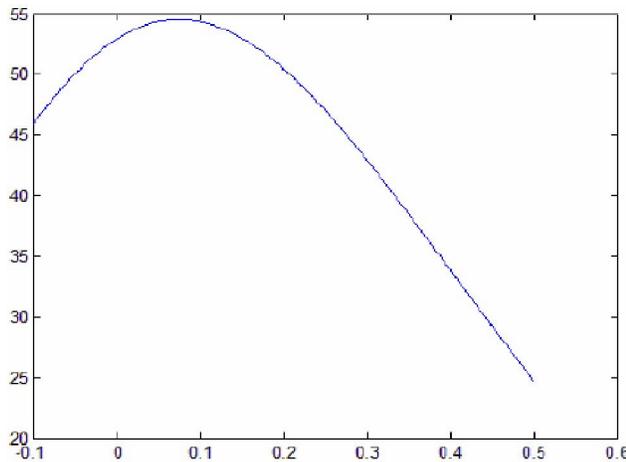


Figure 3 : Hitting speed as well as hitting point to mass center distance relationship diagram

Only consider hands acting force best hitting point model

Investigation shows baseball player sports injury rate has already arrived at as high as 56%, and above 40% players' injury concentrates on hands and shoulder, which shows baseball player hitting motions generate certain degree injury on their body. Based on that, we make research with baseball bat best hitting point as entry point, by establishing model; it plays protection roles on hands to the greatest degree<sup>[4]</sup>.

According to moment of momentum theorem, it

has:  $(J_c + ml^2)\alpha = (1+x)Q$ ,  $\alpha = \frac{(1+x)Q}{J_c + ml^2}$

In formula,  $J_c$  is bat to mass center rotational inertia,  $l$  is hands gripping point E to mass center C distance,  $x$  is impacting point D to mass center C distance,  $\alpha$  is bat rotational angular accelerated speed,  $Q$  is ball to bat impacting force, and then:

$$a_c = l\alpha = l \frac{(1+x)Q}{J_c + ml^2}$$

In formula,  $a_c$  is mass center C accelerated speed. According to mass center motion theorem

$$Q - F = ma_c, \quad Q - F = \frac{m(1+x)Ql}{J_c + ml^2},$$

$$F = Q - \frac{m(1+x)Ql}{J_c + ml^2} = \frac{1}{J_c + ml^2} (J_c - mxl)Q \quad (12)$$

In formula:  $F$  is hands to bat impacting force when impacting.

In the following, it needs to solve ball to bat impact force when impacting. Due to consider that hand grip point displacement basically not change that the position is basic fixed, ball to bat impact force can be solved by mechanics of materials. That is: simplify bat into elastic beam model<sup>[5]</sup> as Figure 5.

Figure 5-a is calculation model, Figure 5-b is under dynamic load deformation Figure, Figure 5-c is under static load deformation figure; then, according to mechanics of materials knowledge:

$$\Delta_{st} = \frac{Qb^3}{3EI_z} \quad (13)$$

$$K_d = 1 + \sqrt{1 + \left( \frac{1}{1 + m_e / m_1} \right)^2 \frac{v^2}{g\Delta_{st}}} \quad (14)$$

In formula:  $\Delta_{st}$  is static deviation,  $E$  is bat Young modulus,  $I_z$  is beam cross section to neural axis inertial moment,  $K_d$  is dynamic load coefficient,  $m_e$  is bat equivalent mass,  $m_e = 33m_2 / 140$ ,  $m_1$  is ball mass,  $v$  is colliding moment ball speed.

When ball speed is larger, dynamic load coefficient can be simplified into:

$$K_d = \sqrt{1 + \left( \frac{1}{1 + m_e / m_1} \right)^2 \frac{v^2}{g\Delta_{st}}}, \quad \sigma_{st} = \frac{Qb}{W}$$

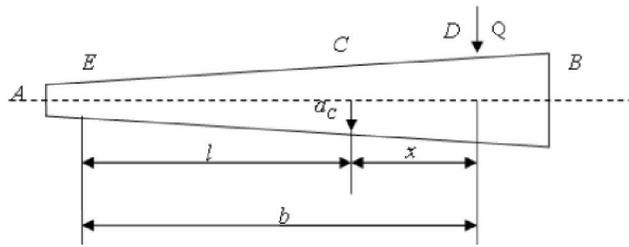
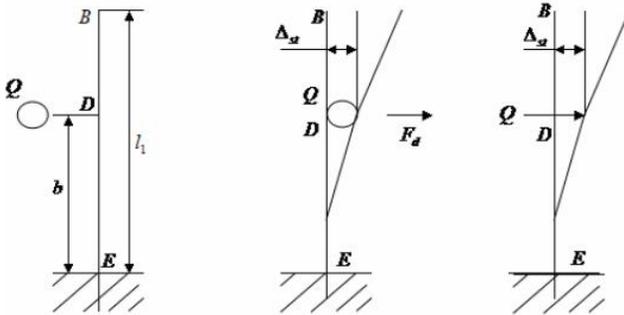


Figure 4: Bat simplified model



a. Calculation model b. Dynamic deformation map loading

Figure 5: Bat elastic beam model simplified diagram

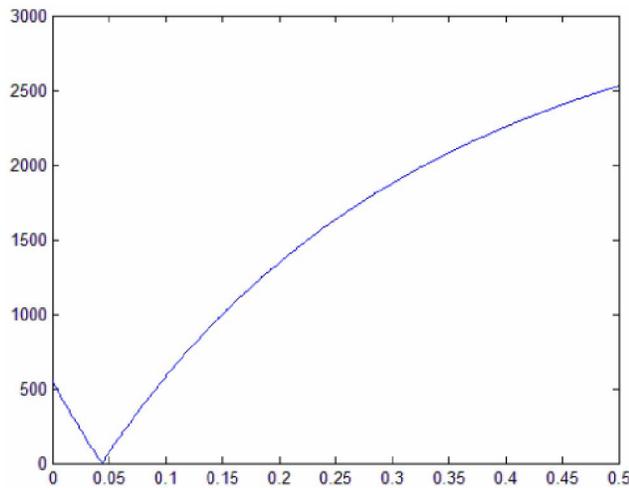


Figure 6 : Bat to hands impact force as well as hitting point to mass center distance relationship figure

In formula:  $W$  is bend section modulus

$$\sigma_d = K_d \sigma_{st} = \frac{Qb}{W} \left( \frac{1}{1 + m_e / m_1} \right) \sqrt{g \Delta_{st} v^2}$$

$$F_d = K_d Q \tag{15}$$

In formula (15),  $\sigma_{st}$  is maximum impact force,  $F_d$  is ball impacting bat moment dynamic load.

Simultaneous formula (12) to (15), it gets opponents' impact force as well as hitting point to mass center distance figure as Figure 6.

From Figure 6, it is clear, with  $x$  increasing,  $F$  value

firstly reduces then increases, by calculation, it can get

when  $x = \frac{J_C}{m_2 l} = 0.0438m$ , bat to hands impact force is

minimum that is 0. At this time, best hitting point to mass center distance is 0.0438m, now bat to hands force is 0.

**Simultaneously consider maximum hitting speed and opponent minimum impact force model**

Because in baseball game, the larger baseball hit speed is, the bigger possibility of athlete scoring would be, to get better results; baseball player should pursuit larger hitting speed. Meanwhile, baseball game injury rate is also higher, which is due to bat to baseball player excessive impact force when hitting will harmful for athletes' hands. So, to ensure baseball players' safety, reduce injury probability, it is required that when players hitting, it pursuits minimum impact force from bat to hands. Simultaneously consider the two statuses; it gives best hitting point mathematical expression as following:

$$Z = F + \lambda / v$$

Among them,  $\lambda$  is weight coefficient. In order to define weight coefficient, we discuss,  $F$  maximum value and minimum value gap are roughly 2500, minimum speed reciprocal and maximum speed reciprocal gap

are roughly  $\frac{1}{25} - \frac{1}{55} = 0.022$ . Therefore we let  $\lambda$  to be

$F = \lambda / v = 113636$ , we define  $= 100000 \cdot Z$  and  $x$  relationship figure is as Figure 7.

Use lingo solving optimal solution, result is as fol-

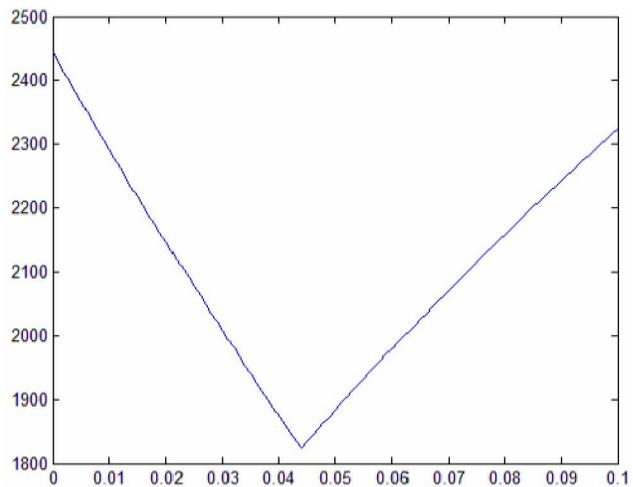


Figure 7 : Z and x relationship diagram

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Variable	Value	Reduced Cost
M1	0.1420000	0.000000
M2	1.400000	0.000000
V1	-40.00000	0.000000
U1	16.00000	0.000000
W1	34.00000	0.000000
I	0.4170000E-01	0.000000
MU	0.000000	0.000000
E	0.4000000	0.000000
X	0.4389474E-01	0.000000

Row	Slack or Surplus	Dual Price
1	1841.438	-1.000000
2	0.4389474E-01	0.000000
3	0.5610526E-01	0.000000

From Figure 7, it is clear when  $x=0.0438\text{m}$ , it gets optimal solution, at this time force  $F=0.918\text{N}$ ,  $v=54.3055\text{m/s}$ .

In the following, discuss  $\lambda$  values influences on results that are the model stability. We discuss optimal solution  $x$  changes with  $\lambda$  changing such relationship as Figure 8.

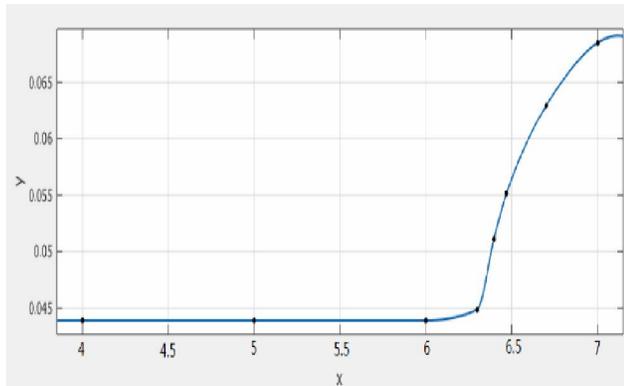


Figure 8 :  $x$  and  $\lambda$  logarithm relationship figure

From Figure 8, it is clear before  $10^6$ , Best hitting point coordinate position basically do not change. By analysis, we thought that in 0.0438 with  $x$  changing,  $F$  values basically not change, which means  $F$  is not sensitive to  $x$ ; While force  $v$  has great changes so that  $v$  is sensitive to  $x$ , it indicates model stability.

## RESULTS AND CONCLUSIONS

From above model, it can conclude that if only con-

sidering maximum hitting speed, when hitting point to mass center distance is 0.074m, speed arrives at maximum that is 54.57m/s. If only considering bat to hands impact force, when hitting point to mass center distance is 0.0438m, bat to hands impact force is the minimum that is 0. If simultaneously considering the two and weight coefficient  $\lambda=100000$ , when  $x=0.0438\text{m}$ , it gets optimal solution, now force  $F=0.918\text{N}$ ,  $v=54.3055\text{m/s}$ , and the model is not sensitive to weight coefficient.

The paper can make expansion to the model from vibration model, adding energy and vibration transferring as well as other aspects factors, and considers hitting process baseball rotation, bat internal stress when hitting and so on, it makes optimization on model.

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