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# Backward surface plasmons in semi-infinite one-dimensional photonic crystals single negative metamaterials

## Abstract

Existence of backward electromagnetic surface waves at an interface separating a semi-infinite uniform left-handed metamaterial and a 1D photonic crystal composed of alternating layers of two kinds of single-negative ($\varepsilon$-negative and $\mu$-negative) metamaterial is theoretically investigated. Dispersion characteristics of surface states are analyzed for two different cases of ENG-MNG and MNG-ENG layered periodic structures. It was demonstrated that in the presence of metamaterial, surface waves are sensitive to light polarization and there exist only backward TM-polarized (or TE-polarized) surface Tamm states depending on the ratio of the thicknesses of two periodic stacking layers.

## INTRODUCTION

Photonic crystals (PCs) attracted intensive studies in the last decade due to their unique electromagnetic properties and potential applications. It has been proven that photonic band gaps (PBGs) could be formed as the result of the interference of Bragg scattering in a periodical dielectric structure. In the conventional PCs (with positive indices), PBGs are highly sensitive to the lattice constant, incident angle and polarization (transverse electric (TE) and transverse magnetic (TM)) of the incident light. The properties of PCs are also affected by disorder, randomness and fabrication tolerances[1]. So, we need some special type of PBGs coming from the mechanisms beyond the Bragg scattering which would be immune to the random thickness error in the fabrication procedure and insensitive to the scale length change, angle of incidence and polarization.

One such attempt is to realize PBG in metamaterials. The metamaterials include double-negative (DNG) materials and single-negative (SNG) materials. DNG materials are artificial composites with both permittivity ($\varepsilon$) and permeability ($\mu$) simultaneously negative and were first theoretically investigated by Veselago in 1968[2–7]. DNG materials exhibit many unusual physical properties different from the conventional right-handed materials. In addition to DNG materials, we can also have SNG materials in which only one of the two material parameters $\varepsilon$ and $\mu$ is negative. The SNG materials consist of $^{2}$-negative (ENG) materials with $\varepsilon < 0$ but $\mu > 0$ and $\mu$-negative (MNG) materials with $\mu < 0$ but $\varepsilon > 0$[8,9]. It is well-known that the electromagnetic wave cannot propagate in ENG or MNG media. However, when ENG and MNG slab are paired in a conjugate manner, some unusual features are exhibited. Al'u and Engheta have given the condition of conjugate matching and shown that such a combination can provide the characteristics of resonance, complete tunneling and transparency, using the equivalent TL modes[10]. Furthermore, the 1D PCs composed of alternate SNG materials can present a new type of PBGs with zero effective phase (denoted as zero-$\phi_{eff}$ gap) that is distinct from the Bragg gaps. Such a zero-$\phi_{eff}$ gap is surrounded by pseudo-propagation modes which are originated from the interaction of forward and backward evanescent waves in the single-negative frequency regime. Zero-$\phi_{eff}$ gap is an omnidirectional gap that is insensitive to the incident angles and polarizations of light. Furthermore, such an omnidirectional gap is invariant with the change of scale length and insensitive to disorder[8]. Surface waves (SWs) have been recognized and studied as a fundamental excitation at the interface between two suitably active media. SWs are typically nonradiative modes propagating along an interface with amplitudes that are evanescent in each bounding medium[11–18]. SWs have become a familiar physical concept in the optics and physics community thanks to the long-history investigation on surface plasmons, which are a kind of localized SWs that are typically excited in metal films. Furthermore, SWs have recently been proposed as a way to efficiently inject light into a PC waveguide, or to extract a focused beam from a channel. In periodic systems, the modes localized at the surfaces are known as Tamm states, first found as localized electronic states at the edge of a truncated periodic potential[19]. SWs generated in PCs have potential to become alternatives to the surface plasmons. They have some advantages. First, SWs supported by PCs can exist virtually in any optical frequency regime due to the scaling nature of dielectric PCs. Second, the low dielectric loss in the structures can lead to sharp resonant coupling between the incoming light and SWs[20]. Several devices have been proposed recently based on the existence of SWs such as optical modulators and sensors[21] and semiconductor laser[22]. Some pplications have been also found for optical communications of the surface modes in PCs for narrow bandpass filters[23]. In this paper, we theoretically study SWs that can be excited at the interfaces between a semi-infinite uniform DNG medium and a semi-infinite 1D PC containing two types of single-negative materials (ENG-MNG and MNG-ENG). We show the excitation of special type of transverse structure for TM-polarized surface waves with a backward energy flow for both ENG-MNG and MNG-ENG periodic structures. Our paper is organized as follows. In Section 2, we present the theoretical model and employ the transfer matrix method (TMM) to calculate SWs at an interface separating a semi-infinite uniform DNG medium and a semi-infinite 1D PC containing SNG materials. Then, in Section 3, the discussion of the dispersion characteristic of the SWs on the plane of the angular frequency versus the propagation constant and existence regions for the backward surface Tamm states are illustrated. Finally, conclusion is given in Section 4.

## MODEL AND NUMERICAL METHODS

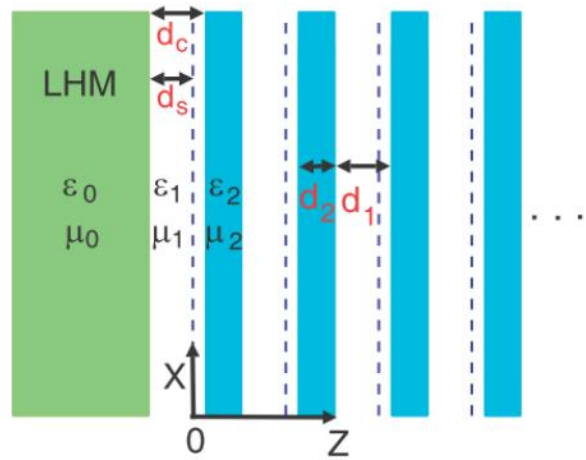


**Figure 1 : Geometry of the problem.**

Let us consider the geometry of the structure, as shown in the Figure 1. We consider a semi- infinite 1D PC consisting of alternating layers of ENG and MNG material. As shown in Figure 1, the PC is capped by a layer of the same SNG material with thickness $d_c$. For the convenience of presentation, we imagine that this cap layer consists of two sub-layers with lengths $d_s$ and $d_t$ respectively, where ($d_s + d_t = d_c$). Then the periodic array that forms the 1D PC consists of cells, each made of three uniform layers of widths dt, d2, and d1-$d_t$.

The permittivity $\varepsilon$ and permeability $\mu$ of the SNG layers have the following forms[8,9],

$$\varepsilon(\omega)=1+\frac{5^2}{0.9^2-\omega^2}+\frac{10^2}{11.5^2-\omega^2}$$
$$\mu(\omega)=1+\frac{3^2}{0.902^2-\omega^2} \tag{1}$$

We take a certain level of the supposed structure to calculate the dispersion properties of the surface modes. For this purpose, we assume that a monochromatic plane wave be incident from air with angle onto the Fibonacci multilayer structure. The electric component and magnetic component can be related via a transfer matrix[7]

$$M_j(\Delta z,\omega)=\begin{pmatrix}\cos(k_z^j\Delta z) & j/q_j\sin(k_z^j\Delta z)\\ jq_j\sin(k_z^j\Delta z & \cos(k_z^j\Delta z)\end{pmatrix} \tag{2}$$

where $q_j=\sqrt{\varepsilon_j}/\sqrt{\mu_j}\sqrt{1-\sin^2/\varepsilon_j\mu_j}$, $k_z^j=\left(\omega/c\right)\sqrt{\varepsilon_j}\sqrt{\mu_j}\sqrt{1-\sin^2/\varepsilon_j\mu_j}$ and c is the speed of light in vacuum, also j = A, B denote ENG and MNG layers, respectively.

## RESULTS AND DISCUSSION

We analyze the dispersion properties of the surface Tamm states in the second SNG gap, so-called zero-$\phi_{eff}$ gap[12], on the plane of the angular frequency $\omega$ versus the propagation constant $\beta$ (see Figure 2) for two different cases of ENG-MNG and MNG-ENG multilayer structures. In contrast to the Bragg gaps, the properties (the central frequency and width of the gap) of such omnidirectional gaps are insensitive to the incident angles and light polarizations, and are invariant upon the change of scale length. As we know, the Tamm states exist in the gaps of the PBG spectrum (unshaded regions in Figure 2). Since our studies show that we have only TM-polarized SWs for proposed structure (see Figure 5), we turn our attention to the TM-polarized SWs and present dispersion characteristic of SWs for diffrent values of the cap layer thickness $d_c$ For ENG-MNG and MNG-ENG multi-layered structures in Figures 2(a) and (b), respectively. It is necessary to remember that in MNG-ENG structure the position of ENG and MNG layers in ENG-MNG structure is exchanged with the same geometry and physical parameters. As one can see from Figure 2, there are different dispersion curves for different values of $d_c$, which describe a possibility to control the dispersion properties of SWs by adjusting dc. Corresponding values of $d_c$ for dotted, dashed, and solid curves of dispersion are $d_c = 0.2d_1$, $0.8d_1$ and $2d_1$, respectively. Moreover, Figure 2 shows that, in ENG-MNG structure, there are no limitation on the existence region of TM surface modes for large dc, whilst in the case of MNG-ENG structure the existence region of TM surface modes decreases by increasing the thickness of cap layer dc (see dashed and solid lines in Figure 2(b)).

Profiles of points (1) and (2), situated in Figure 2, are shown in Figure 3, where we plotted the profiles of the mode (1) in the ENG-MNG structure and the mode (2) in MNG-ENG structure. Figure 3 shows that in ENG-MNG arrangement the peak of SWs is located at the interface between the cap layer and photonic crystal. But in MNG-ENG arrangement, the peak of SWs is located at the interface between DNG material and cap layer.

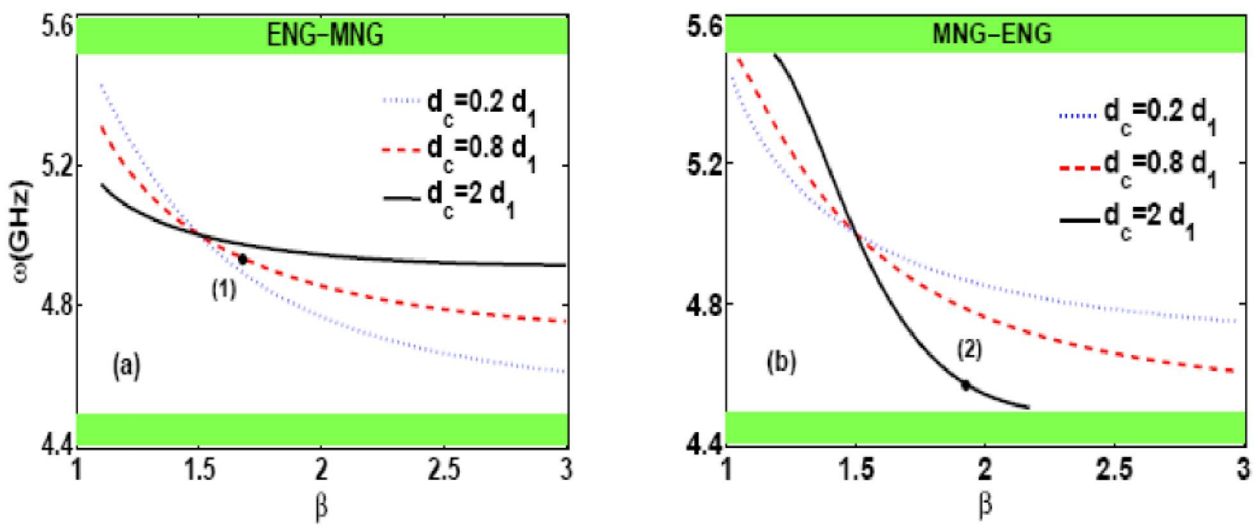


**Figure 2 : Dispersion properties of the TM-polarized surface modes for (a) ENG-MNG and (b) MNG-ENG periodic structures. Unshaded regions show the zero-$\phi_{eff}$ spectral gap of the 1D PC containing SNG materials. Dotted, dashed, and solid curves show the dispersion of the surface modes for $d_c = 0.2d_1$, $0.8d_1$ and $2d_1$, respectively. Points (1), and (2) correspond to the mode profiles presented in Figures 3(a) and 3(b), respectively. The other parameters are the same as the Figure 1.**

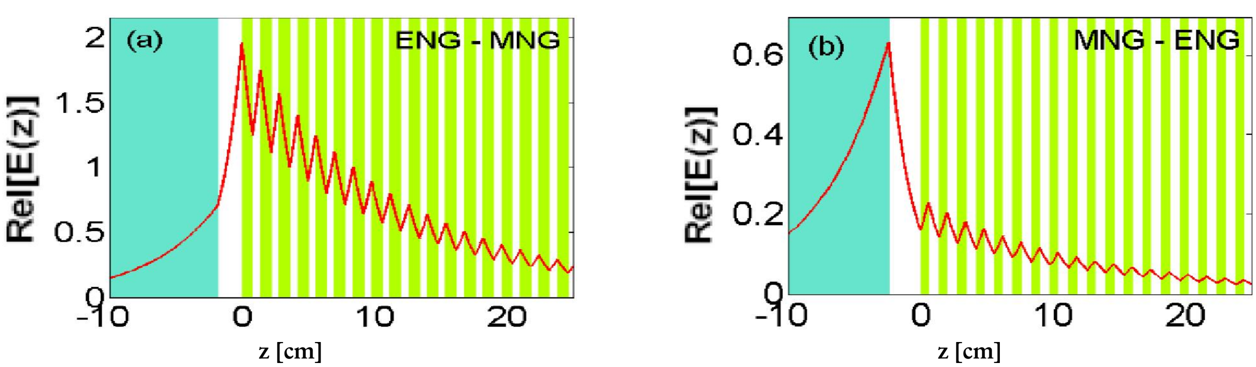


**Figure 3 : Examples of the backward TM surface modes. (a) ENG-MNG structure: $\omega = 4.6$ GHz, $\beta = 1.85$, and $d_c = 0.8d1$. (b) MNG-ENG structure: $\omega = 4.562$ GHz, $\beta = 1.94$, and $d_c = 2d1$. Modes (a), and (b) correspond to the points (1) and (2) in Figure 2, respectively. The other parameters are the same as the Figure 1.**

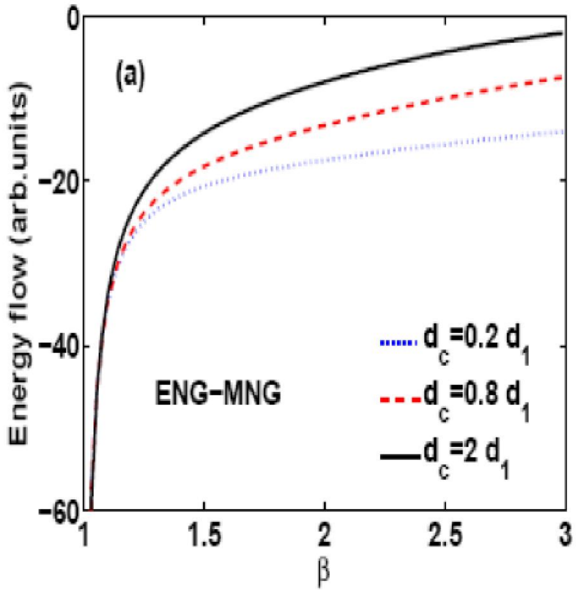


**Figure 4 : Total energy flow of surface Tamm modes vs β in the (a) ENG-MNG and (b) MNG-ENG periodic structures for different $d_c$. Dotted, dashed, and solid curves show the energy flow of the surface modes for $d_c$ = 0.2$d_1$, 0.8$d_1$, and 2$d_1$, respectively.**

Energy flow of surface Tamm states for ENG-MNG and MNG-ENG structure (see Figure 1) has the same behavior as the negative values. To demonstrate this, in Figure 4, we plot total energy flow as a function of the wave number β. We see from Figure 4 that all surface modes in ENG-MNG and MNG-ENG periodic structure have negative energy flow, thus they are backward for different values of cap layer thicknesses $d_c$.

Finally, in Figure 5, we study the dependence of surface modes on the ratio of the thicknesses of two SNG layers (d2/d1) for (a) ENG-MNG (b) MNG-ENG structures. The dotted and solid lines correspond to the surface modes with dc = 0.2d1 and $d_c$= 1.5$d_1$, respectively. Here, β = 1.9, and the other parameters are the same as those in Figure 1. The unshaded regions are omnidirectional zero-$\phi_{eff}$ PBG in which the existence of TM or TE surface modes are indicated. As one can see from Figure 5, in ENG-MNG structure, TE-polarized SWs exist only for the relative thickness ($d_2/d_1$) less than one, and TM-polarized SWs exist only for the relative thickness more than one, while in the case of MNG-ENG structure TE-polarized SWs exist only for the relative thickness ($d_2/d_1$) more than one, and TM-polarized SWs exist only for the relative thickness less than one.

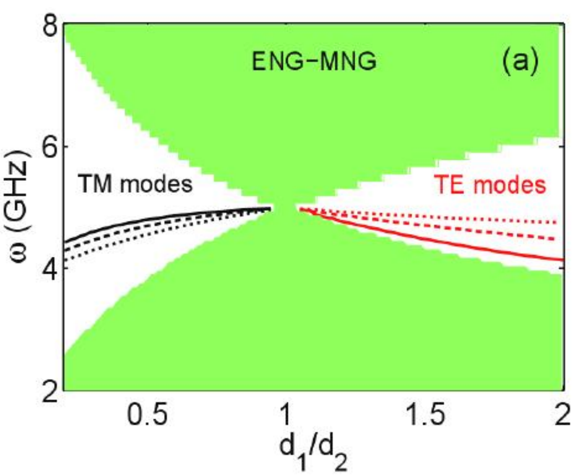


**Figure 5 : The dependence of the zero-$\phi_{eff}$ PBG and surface modes on the ratio of $d_2/d_1$ for (a) ENG-MNG (b) MNG-ENG structures. The dotted and solid lines correspond to the surface modes with $d_c$ = 0.2$d_1$ and $d_c$ = 1.5$d_1$, respectively. Here, β = 1.9, and the other parameters are the same as the Figure 1.**

## CONCLUSION

We have presented a theoretical study of TM-polarized (or TE-polarized) electromagnetic surface waves supported by an interface between a left-handed metamaterial and 1D PC containing alternative ENG-MNG or MNG-ENG layers. We have demonstrated that in the presence of a LHM there are only backward TM-polarized (or TE-polarized) surface Tamm states. Also, we demonstrated that the occurrence of the TE- and TM-polarized surface waves depends on the ratio of the thicknesses of the SNG layers. When the ratio of two SNG layers is less than one (or more than one) in ENG-MNG structure we face the TE (TM) surface waves and in MNG-ENG structure we face the TM (TE) surface waves.

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