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Analysis of dealing strategies and coordination of closed-loop supply chain with differential price under production cost disruptions

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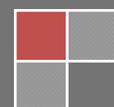
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ABSTRACT

Aimed at a manufacturer collecting closed-loop supply chain (CLSC) with differential price between new and remanufactured products. The coordination of decentralized CLSC is studied considering that production costs of the two products are disrupted. The numerical results show that: The adjustment directions of the normal environment optimal decisions of new and remanufactured products are not only related to the disrupting degree of production cost, but also to the substitute coefficient of two products. The optimal profit of the whole system will be reduced with the increase of production cost, as increased with the reduction of production cost. The improved quantity discount contract could coordinate the decentralized CLSC under disruptions.

KEYWORDS

Manufacturer collecting closed-loop supply chain; Production cost disruptions; Differential price; Coordination; Quantity discount contract.



INTRODUCTION

In recent years, as the resource shortage and environmental contamination problems become increasingly serious, the government of many countries consecutively published various rules and regulations (such as the Management Order for Waste Electric Appliance and Electronic Products issued by EU in 2003 and the Management Order for Recycling of Waste Electric Appliance and Electronic Products implemented by China in 2011) to require the enterprise to be responsible for the product during its entire life cycle, so as to save energy and protect the environment. Meanwhile, with growth of the consumer's environmental awareness, the products manufactured by manufacturers with a good image in terms of environmental protection are more popular. As such, many enterprises have started the reverse logistics actions including recycling of waste products. Those enterprises found during operation that recycling of products could not only help cut down the production cost, reduce consumption of materials, dig the residual value of waste products and drive up economic profit of the enterprise, it can also improve the healthy and green image of the enterprise, improve the relation between the enterprise and consumers in such a fierce competition environment, thus strengthening the economic and social benefit of the enterprise. Hence, due to the impact of the above aspects, the closed-loop supply chain (CLSC) for recycling of waste products has become the focus of the academic circle and business cycle of supply chain management field^[1-3].

Savaskan et al.^[4] divided closed-loop supply chain into recycling by the manufacturer, recycling by the dealer and recycling by the third party depending on the recycling channels. They assumed that the recovery is the decision variable, and the recovery price is the external variable, and concluded that the distributed decision-making closed-loop supply chain of recycling by the dealer is the optimal one in terms of economic and environmental benefit; Savaskan et al.^[5] discussed the recovery channel selection for closed-loop supply chain composed of 2 competing dealers. Afterwards, Gu et al.^[6] assumed that the recovery is not a decision variable but a function of the recovery price, and concluded that the closed-loop supply chain for recycling by dealer was the optimal one. Therefore, the decision variables and objective functions may differ with the aspect from which the closed-loop supply chain is studied, which would lead to different conclusions. However, the above studies all showed that due to the "double marginalization", the benefit of closed-loop supply chain for distributed decision making would cause loss when compared with the closed-loop supply chain for centralized decision making. For this reason, many domestic and foreign scholars studied the closed-loop supply chain for distributed decision making subject to contract coordination. For instance, Savaskan^[4] applied two charge contracts to coordinate the closed-loop supply chain for recycling by dealers; Karakayli et al.^[7] applied two pricing contracts to coordinate the closed-loop supply chain for distributed decision-making for recycling by the third party under the two market right structures including manufacturer as the leader and the third party as the leader; Chen et al.^[8] considered that the consumer may return the product to the dealer due to dissatisfaction and discussed the use of two recycling price buy-back contracts to coordinate the distributed decision making supply chain composed of a manufacturer and a dealer. Such a supply chain mode is similar to the closed-loop supply chain for recycling by the dealer. In fact, due to limit by the re-manufacturing technical level and requirements of laws and regulations (for instance, the Management Order for Recycling of Waste Electric Appliance and Electronic Products of China stipulated that the enterprises should distinguish the new products and the re-manufactured products), and the public's prejudice over the re-manufactured products, there is a price difference on the market between the re-manufactured products and new products and generally the price of the former is lower than that of the latter^[9, 10]. As such, ZhengKejun^[11] respectively used the franchise fee contract and revenue sharing contract to study the closed-loop supply chain for distributed decision making for the price difference between re-manufactured products and the new products.

However, in the above studies, the default was that the closed-loop supply chain made the decision in a stable environment. That is to say, the internal environment of the system is not going to change. However, emergencies, such as natural disasters, macro policy change, terrorist attack, and emergency arising during operation of the enterprise (such as production accident, product failure and employee strike) will severely impact the normal operation of the supply chain. Therefore, the interference of such emergencies with the management of supply chain has drawn large attention from the academic circle. Many scholars carried out studies on how to design contract coordination of the supply chain under interference of such emergencies. For instance, Qi et al.^[12] was the first one to consider the interference of emergency with market demand, and studied the problems with coordination of supply chain using quantity discount contract; soon afterwards, Xu et al.^[13] considered the linear correlation and non-linear correlation between the market demand and the sales price with emergencies impacting the production cost, and studied the problems with coordination of supply chain using quantity discount contract; Xiao et al.^[14] discussed the problem with use of the all unit quantity discount contract and incremental unit quantity discount contract to coordinate the supply chain with emergencies impacting both the market demand and production cost; Zhang et al.^[15] studied the problem with use of the revenue sharing contract to coordinate supply chain for distributed decision making with emergencies respectively interfering the market demand of one dealer and interfering the market demand of both dealers for supply chain composed of one manufacturer and two dealers; Huang et al.^[16] considered the condition in which the emergency interfered with the production cost and studied the problems with pricing and production decision making in the distributed decision making and centralized decision making system for double-channel supply chain composed of a manufacturer and a dealer and pointed out the content worth discussing for double-channel supply chain for distributed decision making with interference of contract coordination emergencies. Based on the above traditional supply chain study results, Wang Yuyan^[17] adopted the revenue sharing contract and MouZongyu et al.^[18] adopted two charging contracts to coordinate the closed-loop supply chain for recycling by the dealer with emergencies interfering with the market demand; Wang Yuyan^[19] carried out a study on the use of quantity discount contract to coordinate

the closed-loop supply chain for recycling by the dealer with emergencies interfering with both the market demand and the production cost; Wang Xu et al. [20] carried out a study on the use of quantity discount contract to coordinate the closed-loop supply chain for recycling by the dealer with emergencies interfering with the market demand and sensitivity coefficient of the recycling price of waste products.

As implied above, the problem with management of enclosed-loop supply chain coordinated by contract under interference of emergencies is a topic to be developed. The previous studies were all carried out on the closed-loop supply chain for recycling by dealers and did not take the difference of price between new products and re-manufactured products. Xerox Corporation saved raw material cost of about 200 million US dollars through recycling and remanufacturing printers within less than 5 years and is regarded as the successful model in management of closed-loop supply chain for recycling by the manufacturer. In addition, Hewlett Packard and Canon both achieve tremendous success by adopting the closed-loop supply chain management mode for recycling by the manufacturer. Hence, to discuss the use of contract in the closed-loop supply chain for recycling by manufacturer to coordinate and deal with the problem with the interference of the emergency, this paper proposed a closed-loop supply chain model for recycling by manufacturer composed of a manufacturer and a dealer. In this model, the pricing difference between new products and remanufactured products is taken into consideration. There is price completion between the two in the consumer market. The following work has been gradually carried out: (1) A quantity discount contract was designed to coordinate the closed-loop supply chain for distributed decision making in the stable environment; (2) The coping strategy of the centralized decision making closed-loop supply chain during discussion of the emergencies interfering with the production cost of the new products and remanufactured products; (3) The problem with use of quantity discount contract to coordinate the distributed decision making closed-loop supply chain with interference of emergencies was analyzed.

BASIC CONTRACT COORDINATION MODEL

The system is a closed-loop supply chain made up of one manufacturer and one dealer; there is information symmetry between the manufacturer and the dealer and both know the cost and demand of each other well; there is a Stan Kohlberg game relationship between the two and the manufacturer acts as the leader while the dealer acts as the follower; the manufacturer recovers waste products and uses the raw material to produce new products and use the recovered waste products to manufacturer the remanufactured products; there is price competition between the new products and remanufactured products in the sales market.

The consumption and symbols in the model are defined as follows:

(1) Assume the waste products of one unit can only be used to produce the remanufactured products of one unit, to avoid additional storage and disposal cost due to surplus of waste products, the manufacturer decides the recovery quantity of waste products based on the market demand for remanufactured products, which means the quantity of waste products should be equal to the quantity of remanufactured products sold on the market. Assume the unit production cost of the remanufactured products made from waste product is, and the unit production cost of new products made from raw material is c_n , then to ensure a profit for the manufacturer to recycle the waste products and remanufacture the products, the following condition should be met: $c_n > c_r > 0$.

(2) Assume the new products and remanufactured products are priced differently, and the manufacturer sales the two types of products to the dealers at the wholesale price of respectively w_n and w_r .

(3) The dealers then sell the new products and remanufactured products at the price of respectively p_n and p_r as shown in Reference [16]. To facilitate analysis, assume the market demand functions of the two types of products are respectively $q_n = \varphi_n - p_n + \gamma p_r$ and $q_r = \varphi_r - p_r + \gamma p_n$. Specifically, φ_n and φ_r respectively represents the maximum market demand scales of new products and remanufactured products. Considering the lack of recognition of remanufactured products among Chinese consumers, the market scale of remanufactured products will be small. Let $\varphi_n > \varphi_r$, and $\gamma (0 \leq \gamma < 1)$ is the substitute coefficient of the two types of products and reflects the mutual substitution degree between new products and remanufactured products.

(4) The manufacture recovers the waste products from consumers at the price of A as shown in Reference [4]. Assume the products returned by the consumers have very low residual value and there is not any secondary market for such products. Then let A be the endogenous variable. Considering the economic performance of the remanufacturing process, there will be $c_r + A \leq c_n$.

Based on the above assumptions and symbol description, the profit functions of the manufacturer, the dealer and the entire supply chain are respectively:

$$\pi_M(w_n, w_r) = (w_n - c_n)(\varphi_n - p_n + \gamma p_r) + (w_r - c_r - A)(\varphi_r - p_r + \gamma p_n) \tag{1}$$

$$\pi_R(p_n, p_r) = (p_n - w_n)(\varphi_n - p_n + \gamma p_r) + (p_r - w_r)(\varphi_r - p_r + \gamma p_n) \tag{2}$$

$$\pi_T(p_n, p_r) = (p_n - c_n)(\varphi_n - p_n + \gamma p_r) + (p_r - c_r - A)(\varphi_r - p_r + \gamma p_n) \tag{3}$$

It could be easily obtained that $\pi_T(p_n, p_r)$ is a strict concave function of p_n and p_r . Therefore, it could be known according to the first-order optimal condition of Formula (3) that: the optimal sales price of new products is

$$p_n^{c*} = \frac{\varphi_n + \gamma\varphi_r + c_n}{2(1-\gamma^2)} + \frac{c_n}{2} \text{ and that of remanufactured product is } p_r^{c*} = \frac{\varphi_r + \gamma\varphi_n + c_r + A}{2(1-\gamma^2)} + \frac{A}{2}.$$

The optimal profit that can be obtained in the entire supply chain is. $\pi_T^* = \frac{[\varphi_n + \gamma\varphi_r - (1-\gamma^2)c_n](\varphi_n - c_n + \gamma c_r + \gamma A)}{4(1-\gamma^2)} + \frac{[\varphi_r + \gamma\varphi_n - (1-\gamma^2)(c_r + A)](\varphi_r - c_r - A + \gamma c_n)}{4(1-\gamma^2)}$

In the distributed decision making supply chain with manufacturer as the Stan Kohlberg game leader, the manufacturer will be the first to make the decision on wholesales pricing of new products and remanufactured products. The dealer will make its own sales price decision of its own new products and remanufactured products following the manufactured. As such, through backward induction, it is easily known that when the manufacturer sells new products and remanufactured products to the dealers at the optimal wholesales price of $w_n^{d*} = \frac{\varphi_n + \gamma\varphi_r + c_n}{2(1-\gamma^2)} + \frac{c_n}{2}$ and $w_r^{d*} = \frac{\varphi_r + \gamma\varphi_n + c_r + A}{2(1-\gamma^2)} + \frac{A}{2}$

respectively, the dealer will sell new products and remanufactured products to consumers at the optimal sales price of $p_n^{d*} = \frac{3(\varphi_n + \gamma\varphi_r) + c_n}{4(1-\gamma^2)} + \frac{c_n}{4}$ and $p_r^{d*} = \frac{3(\varphi_r + \gamma\varphi_n) + c_r + A}{4(1-\gamma^2)} + \frac{A}{4}$ respectively. In such a case, the optimal profit that can be obtained in a

distributed decision making supply chain is $\pi_T^{d*} = \frac{3}{4} \pi_T^{c*}$.

By comparing the optimal results and profit of centralized decision making and distributed decision making supply chain, it is easily known that $p_n^{d*} > p_n^{c*}$; $p_r^{d*} > p_r^{c*}$; $\pi_T^{d*} < \pi_T^{c*}$.

This is because that in the distributed decision making, the manufacturer and the dealer will both pursue the individual profit maximization, which will result in an optimal sales price of new products and remanufactured products higher than the optimal sales price of those two types of products in the centralized decision making supply chain, and further result in a market demand for those two types of products lower than that in the centralized decision making supply chain. Eventually, the optimal profit that can be obtained in the distributed decision making supply chain will be less than that obtained in the centralized decision making supply chain. This is what we call double marginalization.

If through coordination using the contract, the manufacturer allows the optimal decision making of dealer in the distributed decision making supply chain to be equal to that in the centralized decision making supply chain, then the optimal profit that can be realized in the entire distributed decision making supply chain will be equal to that in the centralized decision making supply chain. Then the distributed decision making supply chain is coordinated by the contract. A sufficient condition for the contract to coordinate the supply chain is given in Reference^[16].

Lemma 1: One contract could be used to coordinate the distributed decision making supply chain when: it allows the profit function of the dealer to be the affine (linear) function of the profit function of the centralized decision making supply chain, i.e., $\pi_R = \lambda\pi_T + \eta$ ($\lambda(0 < \lambda < 1)$ and η are both constants).

Below, the problem with use of quantity discount contract to coordinate distributed decision making supply chain will be analyzed. To reflect the quantity of optimal new products and remanufactured products to be purchased by he dealer specified in the contract, $p_n = \frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1-\gamma^2}$ and $p_r = \frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1-\gamma^2}$ is drawn from $q_n = \varphi_n - p_n + \gamma p_r$, and $q_r = \varphi_r - p_r + \gamma p_n$. Introduce them into Formulas (2) and (3), the decision variables of the profit function of the dealer and the entire supply chain could be converted into q_n and q_r as shown below:

$$\pi_R(q_n, q_r) = \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1-\gamma^2} - w_n \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1-\gamma^2} - w_r \right] q_r \tag{4}$$

$$\pi_T(q_n, q_r) = \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1-\gamma^2} - c_n \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1-\gamma^2} - c_r - A \right] q_r \tag{5}$$

Let the quantity discount provided by the dealer be: $T(w_n, q_n, w_r, q_r) = w_n q_n + w_r q_r$ (6)

Then the profit function of the dealer under coordination of quantity discount contract is equal to Formula (4).

Theorem1: For any constant $\lambda(0 < \lambda < 1)$, the quantity discount contract is $T(w_n^*, q_n, w_r^*, q_r) = w_n^* q_n + w_r^* q_r$. Where, when $w_n^* = \lambda \frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} + (1 - \lambda)c_n$ and $w_r^* = \lambda \frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} + (1 - \lambda)(c_r + A)$, it can be used to coordinate the distributed decision making supply chain, and the dealer and manufactured could determine the value of constant λ by bargaining to distribute the optimal profit in the entire supply chain randomly.

Demonstration: By comparing (4) and (5), it can be drawn that:

When $\lambda(0 < \lambda < 1)$ and the w_n^* and w_r^* in the quantity discount contract $T(w_n^*, q_n, w_r^*, q_r) = w_n^* q_n + w_r^* q_r$ are introduced into Formula (4), then we have:

$$\begin{aligned} \pi_R(q_n, q_r) &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - w_n^* \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - w_r^* \right] q_r \\ &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - \lambda \frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - (1 - \lambda)c_n \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - \lambda \frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - (1 - \lambda)(c_r + A) \right] q_r \\ &= (1 - \lambda)\pi_T(q_n, q_r) \end{aligned} \tag{7}$$

This means that the profit function of the dealer is the affine function of the profit function of the entire closed-loop supply chain. As implied by Lemma 1, the quantity discount contract realized coordination of the distributed decision making supply chain. Lemma 1 hereby obtains a proof.

ANALYSIS OF COPING STRATEGY FOR CENTRALIZED DECISION MAKING SUPPLY CHAIN UNDER INTERFERENCE OF PRODUCTION COST

Before occurrence of emergency, the dealer has arranged production of new products of quantity q_n^{c*} and remanufactured products of quantity q_r^{c*} according to the predicted market demand. After occurrence of emergency, the production cost of new products and remanufactured products changed respectively to $c_n + \Delta c_n$ and $c_r + \Delta c_r$. Then obviously, $c_r + \Delta c_r > 0$ only makes sense when $c_n + \Delta c_n > 0$. If the production plan includes production of new products of quantity q_n and remanufactured products of quantity q_r after occurrence of emergency, then the changes of production quantity of new products and remanufactured products when compared with those before the occurrence of emergency are respectively $\Delta q_n = q_n - q_n^{c*}$ and $\Delta q_r = q_r - q_r^{c*}$. Assume $\Delta q_n > 0$ and $\Delta q_r > 0$, for newly increased production of new products and remanufactured products of quantities Δq_n and Δq_r , adjustment of production plan will result in additional unit production cost $\lambda_{11}(0 < \lambda_{11} < c_n)$ and $\lambda_{12}(0 < \lambda_{12} < c_r)$; if $\Delta q_n < 0$ and $\Delta q_r < 0$, for the new products and remanufactured products of quantities $-\Delta q_n$ and $-\Delta q_r$ in excess, there will be additional unit handling cost $\lambda_{21}(0 < \lambda_{21} < c_n)$ and $\lambda_{22}(0 < \lambda_{22} < c_r)$. Therefore, after occurrence of emergencies, the profit function for centralized decision making supply chain is as follows:

$$\begin{aligned} \bar{\pi}_T(q_n, q_r) &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - c_n - \Delta c_n \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - c_r - \Delta c_r - A \right] q_r - \lambda_{11}(q_n - q_n^{c*})^+ - \lambda_{12}(q_r - q_r^{c*})^+ \\ &\quad - \lambda_{21}(q_n^{c*} - q_n)^+ - \lambda_{22}(q_r^{c*} - q_r)^+ \end{aligned} \tag{8}$$

Where, $(x)^+ = \max\{x, 0\}$, and $\lambda_{11}(q_n - q_n^{c*})^+$ and $\lambda_{12}(q_r - q_r^{c*})^+$ respectively represent the additional production cost resulting from increased production of new products and remanufactured products; $\lambda_{21}(q_n^{c*} - q_n)^+$ and $\lambda_{22}(q_r^{c*} - q_r)^+$ respectively represent the additional handling cost resulting from handling of surplus new products and remanufactured products.

By Formula (8): $A' = \frac{\partial^2 \bar{\pi}_T(q_n, q_r)}{\partial q_n^2} = -\frac{2}{1 - \gamma^2}$, $B' = \frac{\partial^2 \bar{\pi}_T(q_n, q_r)}{\partial q_n \partial q_r} = -\frac{2\gamma}{1 - \gamma^2}$, and $C' = \frac{\partial^2 \bar{\pi}_T(p_n, p_r)}{\partial p_r^2} = -\frac{2}{1 - \gamma^2}$. Because $A' < 0$, $A'C' - B'^2 = 4 > 0$, the Hessian matrix $\begin{bmatrix} A' & B' \\ B' & C' \end{bmatrix}$ of function $\bar{\pi}_T(q_n, q_r)$ will be negative definite. It is a strict concave function and has a unique optimal solution.

During real operation, emergencies will often interfere and result in change of the production cost of both new products and remanufactured products towards the same direction. Thus, this paper discusses the situation where the production cost of both new products and remanufactured products change towards the same direction due to interference of

the emergency, i.e., the symbols of Δc_n and Δc_r are the same. Assuming after occurrence of emergency, the optimal output of new products and remanufactured products are respectively $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$, then the following theorem will be made:

Theorem 2: When the production cost of both new products and remanufactured products change towards the same direction due to interference of the emergency, then in the centralized decision making supply chain: 1) When $\Delta c_n \geq 0$ and $\Delta c_r \geq 0$, the optimal output $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$ will not be as follows a) $\bar{q}_n^{c^*} > q_n^{c^*}$, $\bar{q}_r^{c^*} > q_r^{c^*}$; b) $\bar{q}_n^{c^*} = q_n^{c^*}$, $\bar{q}_r^{c^*} > q_r^{c^*}$; c) $\bar{q}_n^{c^*} > q_n^{c^*}$, $\bar{q}_r^{c^*} = q_r^{c^*}$; 2) When $\Delta c_n < 0$ and $\Delta c_r < 0$, the optimal output $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$ will be as follows: d) $\bar{q}_n^{c^*} < q_n^{c^*}$, $\bar{q}_r^{c^*} < q_r^{c^*}$; e) $\bar{q}_n^{c^*} = q_n^{c^*}$, $\bar{q}_r^{c^*} < q_r^{c^*}$; f) $\bar{q}_n^{c^*} < q_n^{c^*}$, $\bar{q}_r^{c^*} = q_r^{c^*}$.

Demonstration: first demonstration 1) When $\Delta c_n \geq 0$ and $\Delta c_r \geq 0$; the reduction to absurdity method is used below:

Assume the optimal output of new products and remanufactured products under interference of emergencies is as follows: a) $\bar{q}_n^{c^*} > q_n^{c^*}$, $\bar{q}_r^{c^*} > q_r^{c^*}$. Under the stable environment, by Formula (5), for when $q_n \geq 0$, $q_r \geq 0$, $\pi_T(q_n, q_r) \leq \pi_T(q_n^{c^*}, q_r^{c^*})$. Thus, when optimal outputs of new products and remanufactured products are respectively $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$, the optimal profit function of centralized decision making supply chain under interference of emergencies is as follows:

$$\bar{\pi}_T(\bar{q}_n^{c^*}, \bar{q}_r^{c^*}) = \pi_T(\bar{q}_n^{c^*}, \bar{q}_r^{c^*}) - \Delta c_n \bar{q}_n^{c^*} - \Delta c_r \bar{q}_r^{c^*} - [\lambda_{11}(\bar{q}_n^{c^*} - q_n^{c^*}) + \lambda_{12}(\bar{q}_r^{c^*} - q_r^{c^*})] \tag{9}$$

Thus, $\bar{\pi}_T(\bar{q}_n^{c^*}, \bar{q}_r^{c^*}) \leq \pi_T(q_n^{c^*}, q_r^{c^*}) - \Delta c_n \bar{q}_n^{c^*} - \Delta c_r \bar{q}_r^{c^*} - \lambda_{11}(\bar{q}_n^{c^*} - q_n^{c^*}) - \lambda_{12}(\bar{q}_r^{c^*} - q_r^{c^*}) < \bar{\pi}_T(q_n^{c^*}, q_r^{c^*})$, which is contradictory with the fact that $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$ are the optimal values of $\bar{\pi}_T(q_n, q_r)$. Therefore, when $\Delta c_n \geq 0$ and $\Delta c_r \geq 0$, the optimal output $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$ will not be as shown in a).

In a similar way, when $\Delta c_n \geq 0$ and $\Delta c_r \geq 0$, the optimal outputs $\bar{q}_n^{c^*}$ and $\bar{q}_r^{c^*}$ of new products and remanufactured products will not be like b) $\bar{q}_n^{c^*} = q_n^{c^*}$, $\bar{q}_r^{c^*} > q_r^{c^*}$ and c) $\bar{q}_n^{c^*} > q_n^{c^*}$, $\bar{q}_r^{c^*} = q_r^{c^*}$.

2) Could find a proof in a similar way as 1). Theorem 2 hereby obtains a proof.

Based on Theorem 2, the following analysis process could be drawn:

Circumstance 1: When $\Delta c_n \geq 0$ and $\Delta c_r \geq 0$, the profit functions for maximized centralized decision making supply chain could be divided into the following binding nonlinear programming problems:

$$(I) \bar{\pi}_T(q_n, q_r) = \left(\frac{\varphi_n + \gamma \varphi_r}{1 - \gamma^2} - \frac{q_n + \gamma q_r}{1 - \gamma^2} - c_n - \Delta c_n\right)q_n + \left(\frac{\varphi_r + \gamma \varphi_n}{1 - \gamma^2} - \frac{q_r + \gamma q_n}{1 - \gamma^2} - c_r - \Delta c_r - A\right)q_r - \lambda_{21}(q_n^{c^*} - q_n) - \lambda_{22}(q_r^{c^*} - q_r),$$

$$(II) \bar{\pi}_T(q_n, q_r) = \left(\frac{\varphi_n + \gamma \varphi_r}{1 - \gamma^2} - \frac{q_n + \gamma q_r}{1 - \gamma^2} - c_n - \Delta c_n\right)q_n + \left(\frac{\varphi_r + \gamma \varphi_n}{1 - \gamma^2} - \frac{q_r + \gamma q_n}{1 - \gamma^2} - c_r - \Delta c_r - A\right)q_r - \lambda_{11}(q_n - q_n^{c^*}) - \lambda_{22}(q_r^{c^*} - q_r),$$

$$(III) \bar{\pi}_T(q_n, q_r) = \left(\frac{\varphi_n + \gamma \varphi_r}{1 - \gamma^2} - \frac{q_n + \gamma q_r}{1 - \gamma^2} - c_n - \Delta c_n\right)q_n + \left(\frac{\varphi_r + \gamma \varphi_n}{1 - \gamma^2} - \frac{q_r + \gamma q_n}{1 - \gamma^2} - c_r - \Delta c_r - A\right)q_r - \lambda_{21}(q_n^{c^*} - q_n) - \lambda_{12}(q_r - q_r^{c^*}),$$

Equations (I) – (III) all meet the following constraint conditions: $q_n^{c^*} - q_n \geq 0$, $q_r - q_r^{c^*} \geq 0$.

Let the optimal solution to (I) be $(\bar{q}_n^{c^*}, \bar{q}_r^{c^*})$. Then the K-T condition is as follows when the Lagrangian multipliers ξ_1 and ξ_2 are introduced:

$$\begin{cases} \partial \bar{\pi}_T(q_n, q_r) / \partial q_n - \xi_1 = 0 \\ \partial \bar{\pi}_T(q_n, q_r) / \partial q_r - \xi_2 = 0 \\ \xi_1(q_n^{c^*} - q_n) = 0 \\ \xi_2(q_r - q_r^{c^*}) = 0 \\ q_n^{c^*} - q_n \geq 0; q_r - q_r^{c^*} \geq 0; \xi_1 \geq 0; \xi_2 \geq 0; q_n \geq 0; q_r \geq 0 \end{cases} \tag{10}$$

The solution to Equation (10) is as follows:

1) When $\Delta c_n \geq \lambda_{21}$, $\Delta c_r \geq \lambda_{22}$, $\Delta c_n \geq \gamma \Delta c_r + \lambda_{21} - \gamma \lambda_{22}$ and $\Delta c_r \geq \gamma \Delta c_n - \gamma \lambda_{21} + \lambda_{22}$, $\bar{q}_n^{c^*} = q_n^{c^*} + \frac{\gamma \Delta c_r - \Delta c_n + \lambda_{21} - \gamma \lambda_{22}}{2}$, $\bar{q}_r^{c^*} = q_r^{c^*} + \frac{\gamma \Delta c_n - \Delta c_r - \gamma \lambda_{21} + \lambda_{22}}{2}$; 2) When $\Delta c_n \geq \lambda_{21}$, $\Delta c_r \geq 0$ and $\Delta c_r < \gamma \Delta c_n + \lambda_{22} - \gamma \lambda_{21}$, $\bar{q}_n^{c^*} = q_n^{c^*} - \frac{(1 - \gamma^2)(\Delta c_n - \lambda_{21})}{2}$, $\bar{q}_r^{c^*} = q_r^{c^*}$; 3) When

$\Delta c_n \geq 0$, $\Delta c_r \geq \lambda_{22}$ and $\Delta c_n < \gamma \Delta c_r + \lambda_{21} - \gamma \lambda_{22}$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*} - \frac{(1-\gamma^2)(\Delta c_r - \lambda_{22})}{2}$; 4) When $0 \leq \Delta c_n < \lambda_{21}$ and $0 \leq \Delta c_r < \lambda_{22}$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$.

In a similar way, the optimal solution to Equation (II) is as follows:

1) When $\Delta c_n \geq 0$, $\Delta c_r \geq \lambda_{22}$ and $\Delta c_n - \gamma \Delta c_r \leq -\lambda_{11} - \gamma \lambda_{22}$, $\bar{q}_n^* = q_n^* + \frac{\gamma \Delta c_r - \Delta c_n - \lambda_{11} - \gamma \lambda_{22}}{2}$, $\bar{q}_r^* = q_r^* + \frac{\gamma \Delta c_n - \Delta c_r + \gamma \lambda_{11} + \lambda_{22}}{2}$; 2)

When $\Delta c_n \geq 0$, $\Delta c_r \geq \lambda_{22}$ and $\Delta c_n > \gamma \Delta c_r - \lambda_{11} - \gamma \lambda_{22}$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*} - \frac{(1-\gamma^2)(\Delta c_r - \lambda_{22})}{2}$; 3) When $\Delta c_n \geq 0$ and $0 \leq \Delta c_r < \lambda_{22}$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$.

The optimal solution to Equation (III) is as follows:

1) 当 $\Delta c_n \geq \lambda_{21}$, $\Delta c_r \geq 0$ 且 $\Delta c_r \leq \gamma \Delta c_n - \gamma \lambda_{21} - \lambda_{12}$ 时, $\bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma \Delta c_r - \Delta c_n + \lambda_{21} + \gamma \lambda_{12}}{2}$, $\bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma \Delta c_n - \Delta c_r - \gamma \lambda_{21} - \lambda_{12}}{2}$; 2) 当

$\Delta c_n \geq \lambda_{21}$, $\Delta c_r \geq 0$ 且 $\Delta c_r > \gamma \Delta c_n - \gamma \lambda_{21} - \lambda_{12}$ 时, $\bar{q}_n^{c*} = q_n^{c*} - \frac{(1-\gamma^2)(\Delta c_n - \lambda_{21})}{2}$, $\bar{q}_r^{c*} = q_r^{c*}$; 3) 当 $0 \leq \Delta c_n < \lambda_{21}$ 且 $\Delta c_r \geq 0$ 时, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$.

1) When $\Delta c_n \geq \lambda_{21}$, $\Delta c_r \geq 0$ and $\Delta c_r \leq \gamma \Delta c_n - \gamma \lambda_{21} - \lambda_{12}$, $\bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma \Delta c_r - \Delta c_n + \lambda_{21} + \gamma \lambda_{12}}{2}$, $\bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma \Delta c_n - \Delta c_r - \gamma \lambda_{21} - \lambda_{12}}{2}$; 2)

When $\Delta c_n \geq \lambda_{21}$, $\Delta c_r \geq 0$ and $\Delta c_r > \gamma \Delta c_n - \gamma \lambda_{21} - \lambda_{12}$, $\bar{q}_n^{c*} = q_n^{c*} - \frac{(1-\gamma^2)(\Delta c_n - \lambda_{21})}{2}$, $\bar{q}_r^{c*} = q_r^{c*}$; 3) When $0 \leq \Delta c_n < \lambda_{21}$ and $\Delta c_r \geq 0$ 时, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$,

Circumstance 2: When $\Delta c_n < 0$ and $\Delta c_r < 0$, the profit functions for maximized centralized decision making supply chain could be divided into the following binding nonlinear programming problems:

$$(\square) \bar{\pi}_r(q_n, q_r) = \left(\frac{\varphi_n + \gamma \varphi_r}{1-\gamma^2} - \frac{q_n + \gamma q_r}{1-\gamma^2} - c_n - \Delta c_n \right) q_n + \left(\frac{\varphi_r + \gamma \varphi_n}{1-\gamma^2} - \frac{q_r + \gamma q_n}{1-\gamma^2} - c_r - \Delta c_r - A \right) q_r - \lambda_{11}(q_n - q_n^{c*}) - \lambda_{12}(q_r - q_r^{c*}),$$

$$(\square) \bar{\pi}_r(q_n, q_r) = \left(\frac{\varphi_n + \gamma \varphi_r}{1-\gamma^2} - \frac{q_n + \gamma q_r}{1-\gamma^2} - c_n - \Delta c_n \right) q_n + \left(\frac{\varphi_r + \gamma \varphi_n}{1-\gamma^2} - \frac{q_r + \gamma q_n}{1-\gamma^2} - c_r - \Delta c_r - A \right) q_r - \lambda_{11}(q_n - q_n^{c*}) - \lambda_{22}(q_r^* - q_r),$$

$$(\square) \bar{\pi}_r(q_n, q_r) = \left(\frac{\varphi_n + \gamma \varphi_r}{1-\gamma^2} - \frac{q_n + \gamma q_r}{1-\gamma^2} - c_n - \Delta c_n \right) q_n + \left(\frac{\varphi_r + \gamma \varphi_n}{1-\gamma^2} - \frac{q_r + \gamma q_n}{1-\gamma^2} - c_r - \Delta c_r - A \right) q_r - \lambda_{21}(q_n^* - q_n) - \lambda_{12}(q_r - q_r^*),$$

Equations (iv)-(vi) all meet the following constraint conditions: $q_n^{c*} - q_n \geq 0$, $q_r - q_r^{c*} \geq 0$.

Similar to solution to Equation (I), the optimal solution to (iv) is as follows: 1) When $\Delta c_n \leq -\lambda_{11}$, $\Delta c_r \leq -\lambda_{12}$, $\Delta c_n \leq \gamma \Delta c_r - \lambda_{11} + \gamma \lambda_{12}$ and $\Delta c_r \leq \gamma \Delta c_n + \gamma \lambda_{11} - \lambda_{12}$, 1) $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$, $-\lambda_{11} < \Delta c_n < \lambda_{21}$ and $-\lambda_{12} < \Delta c_r < \lambda_{22}$; $\bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma \Delta c_r - \Delta c_n - \lambda_{11} + \gamma \lambda_{12}}{2}$, $\bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma \Delta c_n - \Delta c_r + \gamma \lambda_{11} - \lambda_{12}}{2}$; 2) When $\Delta c_n \leq -\lambda_{11}$, $\Delta c_r < 0$ and $\Delta c_r > \gamma \Delta c_n + \gamma \lambda_{11} - \lambda_{12}$, $\bar{q}_n^{c*} = q_n^{c*} - \frac{(1-\gamma^2)(\Delta c_n + \lambda_{11})}{2}$, $\bar{q}_r^{c*} = q_r^{c*}$; 3) When $\Delta c_n < 0$, $\Delta c_r \leq -\lambda_{12}$ and $\Delta c_n > \gamma \Delta c_r - \lambda_{11} + \gamma \lambda_{12}$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*} - \frac{(1-\gamma^2)(\Delta c_r + \lambda_{12})}{2}$; 4) When $-\lambda_{11} < \Delta c_n < 0$ and $-\lambda_{12} < \Delta c_r < 0$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$.

The optimal solution to Equation (v) is as follows:

1) When $\Delta c_n \leq -\lambda_{11}$, $\Delta c_r < 0$ and $\Delta c_r \geq \gamma\Delta c_n + \gamma\lambda_{11} + \lambda_{22}$, $\bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n - \lambda_{11} - \gamma\lambda_{22}}{2}$, $\bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r + \gamma\lambda_{11} + \lambda_{22}}{2}$; 2) When $\Delta c_n \leq -\lambda_{11}$, $\Delta c_r < 0$ and $\Delta c_r < \gamma\Delta c_n + \gamma\lambda_{11} + \lambda_{22}$, $\bar{q}_n^{c*} = q_n^{c*} - \frac{(1-\gamma^2)(\Delta c_n + \lambda_{11})}{2}$, $\bar{q}_r^{c*} = q_r^{c*}$; 3) When $-\lambda_{11} < \Delta c_n < 0$ and $\Delta c_r < 0$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$.

The optimal solution to Equation (vi) is as follows:

1) When $\Delta c_n < 0$, $\Delta c_r \leq -\lambda_{12}$ and $\Delta c_n \geq \gamma\Delta c_r + \lambda_{21} + \gamma\lambda_{12}$, $\bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n + \lambda_{21} + \gamma\lambda_{12}}{2}$, $\bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r - \gamma\lambda_{21} - \lambda_{12}}{2}$; 2) When $\Delta c_n < 0$, $\Delta c_r \leq -\lambda_{12}$ and $\Delta c_n < \gamma\Delta c_r + \lambda_{21} + \gamma\lambda_{12}$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*} - \frac{(1-\gamma^2)(\Delta c_r + \lambda_{12})}{2}$; 3) When $\Delta c_n < 0$ and $-\lambda_{12} < \Delta c_r < 0$, $\bar{q}_n^{c*} = q_n^{c*}$, $\bar{q}_r^{c*} = q_r^{c*}$.

By comprehensively considering the optimal solution to the centralized decision making supply chain for the two conditions $\Delta c_n \geq 0$ and $\Delta c_r \geq 0$ and $\Delta c_n < 0$ and $\Delta c_r < 0$, the following Theorem 3 could be obtained: Theorem 3: When there is any production cost disruption of the new products and remanufactured products subject to interference of emergencies, the values of the optimal output and of the new products and remanufactured products in the centralized decision making supply chain will be as follows:

$$\begin{aligned}
 & \left. \begin{aligned}
 & \bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n + \lambda_{21} + \gamma\lambda_{12}}{2} \quad \bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r - \gamma\lambda_{21} - \lambda_{12}}{2} \quad \Delta c_n \geq \lambda_{21} \text{ 且 } \gamma\Delta c_n - \gamma\lambda_{21} - \lambda_{12} \geq \Delta c_r \geq 0 \\
 & \bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n - \lambda_{11} - \gamma\lambda_{22}}{2} \quad \bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r + \gamma\lambda_{11} + \lambda_{22}}{2} \quad \gamma\Delta c_r - \lambda_{11} - \gamma\lambda_{22} \geq \Delta c_n \geq 0 \text{ 且 } \Delta c_r \geq \lambda_{22} \\
 & \bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n - \lambda_{11} - \gamma\lambda_{22}}{2} \quad \bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r + \gamma\lambda_{11} + \lambda_{22}}{2} \quad \Delta c_n \leq -\lambda_{11} \text{ 且 } \gamma\Delta c_n + \gamma\lambda_{11} + \lambda_{22} \leq \Delta c_r < 0 \\
 & \bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n + \lambda_{21} + \gamma\lambda_{12}}{2} \quad \bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r - \gamma\lambda_{21} - \lambda_{12}}{2} \quad \gamma\Delta c_r + \lambda_{21} + \gamma\lambda_{12} \leq \Delta c_n < 0 \text{ 且 } \Delta c_r \leq -\lambda_{12}
 \end{aligned} \right\} 2) \\
 & \left. \begin{aligned}
 & \bar{q}_n^{c*} = q_n^{c*} - \frac{(1-\gamma^2)(\Delta c_n - \lambda_{21})}{2} \quad \bar{q}_r^{c*} = q_r^{c*} \quad \Delta c_n \geq \lambda_{21}, \Delta c_r \geq 0 \text{ 且 } \gamma\Delta c_n - \gamma\lambda_{21} - \lambda_{12} < \Delta c_r < \gamma\Delta c_n - \gamma\lambda_{21} + \lambda_{22} \\
 & \bar{q}_n^{c*} = q_n^{c*} \quad \bar{q}_r^{c*} = q_r^{c*} - \frac{(1-\gamma^2)(\Delta c_r - \lambda_{22})}{2} \quad \Delta c_n \geq 0, \Delta c_r \geq \lambda_{22} \text{ 且 } \gamma\Delta c_r - \gamma\lambda_{22} - \lambda_{11} < \Delta c_n < \gamma\Delta c_r - \gamma\lambda_{22} + \lambda_{21} \\
 & \bar{q}_n^{c*} = q_n^{c*} - \frac{(1-\gamma^2)(\Delta c_n + \lambda_{11})}{2} \quad \bar{q}_r^{c*} = q_r^{c*} \quad \Delta c_n \leq -\lambda_{11}, \Delta c_r < 0 \text{ 且 } \gamma\Delta c_n + \gamma\lambda_{11} - \lambda_{12} < \Delta c_r < \gamma\Delta c_n + \gamma\lambda_{11} + \lambda_{22} \\
 & \bar{q}_n^{c*} = q_n^{c*} \quad \bar{q}_r^{c*} = q_r^{c*} - \frac{(1-\gamma^2)(\Delta c_r + \lambda_{12})}{2} \quad \Delta c_n < 0, \Delta c_r \leq -\lambda_{12} \text{ 且 } \gamma\Delta c_r - \lambda_{11} + \gamma\lambda_{12} < \Delta c_n < \gamma\Delta c_r + \lambda_{21} + \gamma\lambda_{12}
 \end{aligned} \right\} 3) \\
 & \left. \begin{aligned}
 & \bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n + \lambda_{21} - \gamma\lambda_{22}}{2} \quad \bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r - \gamma\lambda_{21} + \lambda_{22}}{2} \quad \Delta c_n \geq \lambda_{21}, \Delta c_r \geq \lambda_{22}, \Delta c_r \geq \gamma\Delta c_n - \gamma\lambda_{21} + \lambda_{22} \text{ 且 } \Delta c_n \geq \gamma\Delta c_r + \lambda_{21} - \gamma\lambda_{22} \\
 & \bar{q}_n^{c*} = q_n^{c*} + \frac{\gamma\Delta c_r - \Delta c_n - \lambda_{11} + \gamma\lambda_{12}}{2} \quad \bar{q}_r^{c*} = q_r^{c*} + \frac{\gamma\Delta c_n - \Delta c_r + \gamma\lambda_{11} - \lambda_{12}}{2} \quad \Delta c_n \leq -\lambda_{11}, \Delta c_r \leq -\lambda_{12}, \Delta c_r \leq \gamma\Delta c_n + \gamma\lambda_{11} - \lambda_{12} \text{ 且 } \Delta c_n \leq \gamma\Delta c_r - \lambda_{11} + \gamma\lambda_{12}
 \end{aligned} \right\} 4)
 \end{aligned}$$

Theorem 3 shows that when the disruption of production cost of new products and remanufactured products when subject to interference of emergencies is not great, as in Circumstance 1, the optimal output of the two types of products should remain unchanged in a stable environment. When the disruption of production cost of one type of products when subject to interference of emergencies is great, and that of the production cost of the other type of products is kept within a certain extent (it is easily obtained that for the extent and the disruption degree of the production cost of the products with great disruption are easily, there is positive correlation between the substitution coefficients of the two types of products), for the products with great disruption, the optimal output in the stable environment should be adjusted towards the direction contrary to the disturbance direction of its production cost due to interference of emergencies; if the disruption degree is small, as in Circumstance (2), the optimal output of the type of products in the stable environment should remain the same; when the disruption degree of the two types of products due to interference of the emergencies are both great, as in Circumstance (4), the optimal outputs of the two types of products in a stable environment should be adjusted towards the direction contrary to the disturbance direction of its production cost due to interference of emergencies.

By $p_n = \frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2}$, $p_r = \frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2}$, and $\bar{\pi}_r(\bar{q}_n^{c*}, \bar{q}_r^{c*})$, the optimal sales price of new products and remanufactured products and the optimal profit of the entire centralized decision making supply chain could be easily obtained.

ANALYSIS OF DISTRIBUTED CLOSED-LOOP SUPPLY CHAIN WITH PRODUCTION COST DISRUPTION COORDINATED BY CONTRACT

As shown in Reference^[12-15], in the distributed decision making supply chain, the additional production and handling cost after occurrence of emergencies will be born by the manufacturer. Therefore, the profit function mode of the dealer in case of disruption by emergencies will be the same as that in a stable environment.

In real cases, the manufacturer will be affected by time, space and information feedback. Some times the manufacturer may not be able to discover the occurrence of emergencies. Therefore, the coordination of distributed decision making supply chain when the manufacturer still uses the quantity discount contract that is used in a stable environment will be analyzed. The following Theorem will obtain a proof.

Theorem 4: After occurrence of emergencies, the quantity discount contract $T(w_n^*, q_n, w_r^*, q_r) = w_n^* q_n + w_r^* q_r$ used in a stable environment can not be used to coordinate the distributed decision making supply chain.

Demonstration: By comparing Equations (4) and (8): After occurrence of emergencies, when w_n^* and w_r^* in the quantity discount contract $T(w_n^*, q_n, w_r^*, q_r) = w_n^* q_n + w_r^* q_r$ are introduced into Equation (4):

$$\begin{aligned} \pi_R(q_n, q_r) &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - w_n^* \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - w_r^* \right] q_r \\ &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - \lambda \frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - (1 - \lambda)c_n \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - \lambda \frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - (1 - \lambda)(c_r + A) \right] q_r \\ &= (1 - \lambda)\bar{\pi}_r(q_n, q_r) + (1 - \lambda)[\Delta c_n q_n + \Delta c_r q_r + \lambda_{11}(q_n - q_n^{c*})^+ + \lambda_{12}(q_r - q_r^{c*})^+ + \lambda_{21}(q_n^{c*} - q_n)^+ + \lambda_{22}(q_r^{c*} - q_r)^+] \end{aligned} \tag{12}$$

By Equation (12), it could be obtained that the profit function of dealer is no longer the affine function of the profit function of the entire supply chain as subject to disruption by emergencies. Hence, the quantity discount contract $T(w_n^*, q_n, w_r^*, q_r) = w_n^* q_n + w_r^* q_r$ can not be used to coordinate the distributed decision making supply chain under disruption by emergencies. Theorem 4 thus obtains a proof.

Theorem 5: For any constant $\lambda(0 < \lambda < 1)$, when the quantity discount contract is improved as

$$T(\bar{w}_n, q_n, \bar{w}_r, q_r) = \bar{w}_n q_n + \bar{w}_r q_r, \text{ where } \bar{w}_n = w_n^* + (1 - \lambda) \left[\Delta c_n + \frac{\lambda_{11}(q_n - q_n^{c*})^+ + \lambda_{21}(q_n^{c*} - q_n)^+}{q_n} \right],$$

$$\bar{w}_r = w_r^* + (1 - \lambda) \left[\Delta c_r + \frac{\lambda_{12}(q_r - q_r^{c*})^+ + \lambda_{22}(q_r^{c*} - q_r)^+}{q_r} \right],$$

it can be used to coordinate distributed decision making supply chain under disruption by emergencies. In addition, the manufacturer and dealer could determine the value of constant parameter λ by bargaining to freely distribute the optimal profit that can be obtained in the entire supply chain under disruption by the emergencies.

Demonstration: By comparing Equations (4) and (8):

For any $\lambda(0 < \lambda < 1)$, when \bar{w}_n and \bar{w}_r in quantity discount contract $T(\bar{w}_n, q_n, \bar{w}_r, q_r) = \bar{w}_n q_n + \bar{w}_r q_r$ are brought into Equation (2):

$$\begin{aligned} \pi_R(q_n, q_r) &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - \bar{w}_n \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - \bar{w}_r \right] q_r \\ &= \left[\frac{\varphi_n - q_n + \gamma(\varphi_r - q_r)}{1 - \gamma^2} - w_n^* - (1 - \lambda)\Delta c_n - (1 - \lambda) \frac{\lambda_{11}(q_n - q_n^{c*})^+ + \lambda_{21}(q_n^{c*} - q_n)^+}{q_n} \right] q_n + \left[\frac{\varphi_r - q_r + \gamma(\varphi_n - q_n)}{1 - \gamma^2} - w_r^* - (1 - \lambda)\Delta c_r - (1 - \lambda) \frac{\lambda_{12}(q_r - q_r^{c*})^+ + \lambda_{22}(q_r^{c*} - q_r)^+}{q_r} \right] q_r \\ &= (1 - \lambda)\bar{\pi}_r(q_n, q_r) \end{aligned} \tag{13}$$

This means the profit function of dealer is also the affine function of profit function of the entire supply chain. As inferred from Theorem 1, the improved quantity discount contract could be used to coordinate the distributed decision making supply chain. Theorem 5 thus obtains a proof.

As inferred from Theorem 5, by using the adjusted quantity discount contract, the manufacturer and dealer could jointly bear the risk related to additional production and handling cost resulting from emergencies. This way, the various members in the distributed decision making supply chain coordinated by a contract could cope with the disruption by emergencies in joint efforts.

When there is no emergency, i.e. $\Delta c_n = 0$ and $\Delta c_r = 0$, the contract parameter values 为 $\bar{w}_n = w_n^*$, $\bar{w}_r = w_r^*$, in the improved quantity discount contract $T(\bar{w}_n, q_n, \bar{w}_r, q_r) = \bar{w}_n q_n + \bar{w}_r q_r$. In such a case, it is the same as the quantity discount contract $T(w_n^*, q_n, w_r^*, q_r) = w_n^* q_n + w_r^* q_r$ in a stable environment. Therefore, the improved quantity discount contract could be used to adjust the distributed decision making supply chain in the stable environment.

ANALYSIS OF EXAMPLES

The conclusion drawn above will be demonstrated through numerical value examples below. Assume the values of various exogenous variables in the model are as follows: $\varphi_n = 25$, $\varphi_r = 15$, $c_n = 25$, $c_r = 15$, $A = 4$, $\gamma = 0.5$.

The sales prices of new products and remanufactured products in the centralized decision making supply chain in a stable environment are $p_n^{c*} = 279.17$, $p_r^{c*} = 242.83$. Thus, the optimal outputs of the two types of products are $q_n^{c*} = 142.25$, $q_r^{c*} = 96.75$, and the optimal profit that can be obtained is $\pi_T^{c*} = 57811.08$.

In the distributed decision making supply chain in a stable environment, if the dealer and the manufacturer determined the value of the quantity discount contract parameter λ as 0.6 through bargaining, then the wholesale prices of new products and remanufactured products in the quantity discount contract are $w_n^* = 177.50$, $w_r^* = 157.30$. It can be used to coordinate the distributed decision making supply chain.

Assume $\lambda_{11} = 2$, $\lambda_{12} = 2$, $\lambda_{21} = 2$ and $\lambda_{22} = 2$.

Based on Theorem, it can be drawn that in case of disruption of the production cost of the new products and remanufactured products to different extents due to interference of emergencies, the optimal value of the centralized decision making supply chain and the optimal profit of the entire system are given in Table 1:

The analysis results of Table 1 indicate that:

Circumstance 1: When the disruption extent of production cost of two types of products due to interference of emergencies is not great, i.e., $\Delta c_n = 1, \Delta c_r = 1$ or $\Delta c_n = -1, \Delta c_r = -1$, the optimal sales price and output of the two types of products in a stable environment should be kept the same to cope with the emergencies effectively.

TABLE 1: Closed-loop supply chain dealing strategies under different production cost disruptions

	Δc_n	Δc_r	\bar{p}_n^{c*}	\bar{p}_r^{c*}	\bar{q}_n^{c*}	\bar{q}_r^{c*}	\bar{w}_n	\bar{w}_r	$\bar{\pi}_T^*$
1	1	1	279.17	242.83	142.25	96.75	177.90	157.70	57572.08
	-1	-1	279.17	242.83	142.25	96.75	177.10	156.90	58050.08
	10	2	283.17	244.83	139.25	96.75	181.52	158.10	56237.08
2	2	10	281.17	246.83	142.25	93.75	178.30	161.33	56601.08
	-10	-2	275.17	240.83	145.25	96.75	173.52	156.50	59469.08
	-2	-10	277.16	238.83	142.25	99.75	176.70	153.32	59105.08
3	10	5	283.17	244.83	139.25	96.75	181.52	159.30	55946.83
	5	10	281.17	246.83	142.25	93.75	179.50	161.33	56174.33
	-10	-5	275.17	240.83	145.25	96.75	173.52	155.30	59759.33
4	-5	-10	277.17	238.83	142.25	99.75	175.50	153.32	59531.83
	10	10	283.17	246.83	140.25	94.75	181.51	161.32	55477.08
	-10	-10	275.17	238.83	144.25	98.75	173.51	153.32	60257.08

Circumstance 2: When the production cost of one type of products increased a lot while that of the other increased a little due to interference of emergencies, i.e. $\Delta c_n = 10, \Delta c_r = 2$ or $\Delta c_n = 2, \Delta c_r = 10$, the optimal policy in a stable environment is to raise the sales price of the two and reduce the output of the products seriously disrupted by the emergency while increasing the output of the products slightly disrupted by the emergency. When the production cost decreased a lot while that of the other decreased a little due to interference of emergencies, i.e. $\Delta c_n = -10, \Delta c_r = -2$ or $\Delta c_n = -2, \Delta c_r = -10$, then the

optimal policy in a stable environment is to reduce the sales price of the two and increase the output of the products seriously disrupted by the emergency while reducing the output of the products slightly disrupted by the emergency.

Circumstance 3: When the production cost of one type of products increased a lot while that of the other increased comparatively largely due to interference of emergencies, i.e. $\Delta c_n = 10, \Delta c_r = 5$ or $\Delta c_n = 5, \Delta c_r = 10$, the optimal policy in a stable environment is to raise the sales price of the two and reduce the output of the products seriously disrupted by the emergency while keeping the output of the other type of products unchanged. When the production cost decreased a lot while that of the other decreased comparatively largely due to interference of emergencies, i.e. $\Delta c_n = -10, \Delta c_r = -5$ or $\Delta c_n = -5, \Delta c_r = -10$, then the optimal policy in a stable environment is to reduce the sales price of the two and increase the output of the products seriously disrupted by the emergency while reducing the output of the other types of products.

Circumstance 4: When production cost of the two types of products is seriously disrupted by the emergencies, and the disruption degrees of the two types of products are both great, i.e. $\Delta c_n = 10, \Delta c_r = 10$ or $\Delta c_n = -10, \Delta c_r = -10$, then the optimal policy in a stable environment is adjust the sales price of the two types of products towards the same direction and adjust the outputs of the two in opposite directions.

In the above 4 circumstances, the optimal profit of the entire system will decrease with increase of the production cost and will increase with decrease of the production cost of the two.

As indicated in Table 1, for the optimal value of the contract parameter in a stable environment, the wholesale prices of the two types of products in the quantity discount contract will decrease with decrease of the production cost due to interference of the emergencies. This indicates that the manufacturer has managed to cope with the interference of emergencies jointly with the dealer through coordination using the quantity discount contract.

CONCLUSION

This paper studied the problems with the use of quantity discount contract to cope with the disruption of production cost of new products and remanufactured products due to interference of emergencies. It proposed the production policies for new products and remanufactured products in various disruption conditions and improved the quantity discount contract in the stable environment to allow it to coordinate the supply chain whether there is any emergency. At last, this paper demonstrated the validity of the drawn conclusions through numerical value examples. The studies showed that:

(1) Quantity discount contract could be used to coordinate the “dual marginalization” of the closed-loop supply chain for recycling by manufacturer when the new products and remanufactured products are priced differently and improve its operation profit.

(2) For centralized decision making supply chain, when there is disruption of production cost of both new products and remanufactured products due to interference of emergencies, the optimal pricing decision making and the production plan of the two types of products in a stable environment have certain robustness; when the production cost of one type of products is disrupted seriously by emergency while that of the other is only disrupted to a certain extent, then, for the former, the optimal sales price in a stable environment should be adjusted towards the same direction as the interference of emergencies and the optimal output in a stable environment should be adjusted towards the opposite direction; in such a case, for the other type of products, the coping strategy depend on the disruption extent of the production cost of the products seriously disrupted by the emergency and the substitution coefficient of the two types of products; when the production cost of both types of products are seriously disrupted by interference of emergencies, the optimal sales price in a stable environment should be adjusted towards the same direction as the interference of the emergency and the optimal output in a stable environment should be adjusted towards the opposite direction. In any of the above conditions, the optimal profit of the entire system will decrease with increase of the production cost of the two types of products and will increase with decrease of the production cost.

(3) By the improved quantity discount contract, the manufacturer and the dealer could cope with the emergencies in joint efforts to coordinate the distributed decision making supply chain with interference of emergencies.

The problems with the use of contract to coordinate the closed-loop supply chain in case of asymmetry of information on the interference of emergencies could be further discussed based on the study findings of this paper.

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