

Adiabatic coupling constant g of the binary liquid mixture methanol – cyclohexane

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ABSTRACT

The dynamic shear viscosity of the binary liquid mixture methanol - cyclohexane for different temperatures and concentrations is measured using digital viscometer with UL adapter. Shear viscosity anomaly is clearly observed near the critical temperature $T_c = 45.2$ °C and the critical concentration $X_c = 30\%$ by weight of methanol. The specific heat at constant pressure of the critical mixture methanol – cyclohexane was calculated using two scale factor universality. The dynamic scaling theory of Ferrell and Bhattacharjee is applied to the data of the ultrasonic

absorption coefficients α_c at different frequencies. The linear relation of $\frac{\alpha_c}{f^2}$ versus $f^{-1.06}$ was obtained. The adiabatic coupling constant g , isobaric thermal expansion coefficient α_p and diffusion coefficient D were

calculated. The experimental values of $\frac{\alpha(X_c, T)}{\alpha(X_c, T_c)}$ were plotted as a function of the reduced frequency ω^* and it showed a good agreement with the theoretical scaling function $F(\omega^*)$ presented by Ferrell and Bhattacharjee.

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INTRODUCTION

The study of ultrasonic attenuation through absorption or dispersion is important to investigate the properties of matter in its three states. The ultrasonic velocity in a medium gives valuable information about the physical characteristics of the medium. Moreover, the ultrasonic absorption has become a powerful tool in providing important information about various inter and intra - molecular processes such as relaxation of the medium or existence of isomeric states^[1].

LIQUID SYSTEMS

There are two types of liquid systems; first is

the pure one which composed of one liquid such as olive oil, benzene methanol or coconut oil. The other one is a mixture that is composed of two or more liquids,^[2]. Binary liquid mixture consists of two liquids that have solubility to each other at a certain temperature called critical temperature and a certain concentration called critical concentration. At the critical temperature and the critical concentration they become as one liquid; such as benzene - coconut oil, methanol - cyclohexane, benzene – tetrachloride and pentanol – nitromethane. Another type of mixtures is called ternary liquid mixture. This type is composed of three different liquids that have solubility to each other at certain concentration and certain temperature^[3].

LITERATURE REVIEW

There are numerous studies which discuss the properties of pure, binary and ternary liquid mixtures using different theories, such as: mode coupling, renormalization or dynamic scaling theories.

Ferrell and Bhattacharjee presented a new theory of critical ultrasonic attenuation in binary liquid mixtures based on the frequency - dependent specific heat. The theoretical results are fitted with the experimental ones^[4].

The acoustic velocity and attenuation have been measured for the binary liquid mixture 3-methylpentane - nitroethane in the frequency range 1 – 17MHz and temperature range $0.09 \leq T - T_c \leq 13.5K$. The experimental data of the reduced frequency fitted with the dynamic scaling, renormalization and mode coupling theories by Garland and Sanchez. The scaling function as a function of the reduced frequency was plotted using the dynamic scaling theory. It is concluded that Ferrell and Bhattacharjee hypothesis of scaling function is in a good agreement with experimental results^[5].

The ultrasonic wave attenuation for triethylamin - water binary liquid mixture was measured according to the dynamic scaling theory at the critical temperature. The relation between ultrasonic absorption coefficient

$\left(\frac{\alpha}{f^2}\right)$ versus $f^{-1.06}$ was proved to be straight line according to the dynamic scaling theory. The adiabatic coupling constant (g) has been evaluated by Fast and Yun^[6].

Jacobs has measured the turbidity of the critical mixture methanol – cyclohexane above its critical point. The correlation length ξ was calculated using the two scale factor universality^[7].

Ferrell found that the sound propagation produce temperature swings if the frequency is smaller than the relaxation time^[8].

Abdelraziq and his group studied the ultrasonic absorption and velocity as a function of temperature and concentration, shear viscosity is studied as a function of concentration and temperature for nitrobenzene-n-hexane above the critical temperature

range between 5 - 25MHz, using the dynamic scaling theory^[9]

Abdelraziq and his team have measured the ultrasonic velocity and absorption for the binary liquid mixture carbon tetrachloride – coconut oil. The dynamic scaling theory was applied in the frequency range of 5 – 35MHz^[10].

The ultrasonic absorption coefficient α was measured for the binary liquid mixture cyclohexane – aniline by Abdelraziq. The dynamic scaling theory was applied in the frequency range of 5 – 35MHz^[11].

Abdelraziq and his group have measured the shear viscosity as a function of temperature for the binary liquid mixture nitrobenzene – n-heptane. The Debye momentum cutoff q_D was calculated using the mode coupling theory^[12].

Abdelraziq studied the ultrasonic absorption at 5 - 25MHz frequency range and velocity measurements above the critical temperature for perfluoromethylcyclohexane - carbon tetrachloride^[13].

The adiabatic coupling constant, correlation length and diffusion coefficient were obtained for the binary mixture aniline - cyclohexane using mode coupling theory by Abdelraziq^[14].

Abdelraziq has measured the shear viscosity for the binary mixture nitroethane – 3-methylpentane in the temperatures range $0.040 \leq T - T_c \leq 18.570K$. Debye momentum cutoff q_D and the constant A have been calculated using the mode coupling theory^[15].

Behrends and his group had tested methanol- cyclohexane binary liquid system in the frame of dynamic scaling theory and the adiabatic coupling constant was calculated. It is concluded that the temperature variation is due to the g temperature dependence and thermal expansion coefficient^[16].

Abdelraziq studied the ultrasonic absorption at 5-25MHz frequency range for benzene-coconut oil and hexane- β , β -dichloroethyl ether mixture at different temperatures above the critical one^[17].

Bhattacharjee and his group discussed the origin of sound attenuation through liquid systems around the critical point. Quasi - elastic scattering, shear viscosity and heat capacity were measured and studied theoretically according to the dynamic scal-

ing theory^[18].

OBJECTIVES OF THE STUDY

The main goal of this work is to apply the dynamic scaling theory for the binary liquid mixture methanol – cyclohexane. Several thermodynamic properties will be determined for the binary mixture under study as critical heat capacity, isobaric thermal expansion coefficient, the adiabatic coupling constant, diffusion coefficient and critical sound velocity.

Theory

This paper is based on the dynamic scaling theory which proposed by Ferrell and Bhattacharjee to study the ultrasonic attenuation in binary liquid mixtures^[4].

In the dynamic scaling theory the total absorption coefficient at the critical temperature and concentration can be simply expressed as^[3,4,13,18].

$$\left| \frac{\alpha(x_c, T_c)}{f^2} = S f^{-1.06} + b \right| \quad (1)$$

where b represents the contribution of the frequency independent background absorption. The S value is given by^[3,4,13,18].

$$\left| S = \left[\frac{\pi^2 C_{pc} g^2 v_c \dot{\alpha}}{2z \gamma T_c C_p^2(t_f)} \right] \left[\frac{a \omega_0}{2\pi} \right]^{\dot{\alpha}/z\gamma} \right| \quad (2)$$

Here $\dot{\alpha} = 0.11$ and $\overline{z \gamma} = 1.9$ are the critical exponents, C_{pc} is the critical amplitude in the following expression for the specific heat at constant pressure of a mixture of critical composition^[3,13,18].

$$\left| C_p = C_{pc} t^{-\dot{\alpha}} + C_{pb} \right| \quad (3)$$

C_{pb} is the background specific heat, a is a dimensionless scaling factor of order unity, is a characteristic temperature-dependent relaxation rate, g is the adiabatic coupling constant, v_c is the adiabatic sound velocity at T_c , and $C_p(t_f)$ is the specific heat at a characteristic reduced temperature t_f , which can be

approximated by $t = \left(\frac{T - T_c}{T_c} \right)$.

The adiabatic coupling constant g was introduced by Ferrell and Bhattacharjee and is given by^[3,4,13,18].

$$\left| g = \rho_c C_p \left(\frac{dT_c}{dP} - \frac{T \alpha_p}{\rho C_p} \right) \right| \quad (4)$$

where is ρ_c the density at critical temperature and concentration and α_p is the isobaric thermal expansion coefficient.

The absorption coefficient $\alpha(\text{crit } \omega, T)$ can also be expressed as a function of the dimensionless reduced frequency ω^* ^[3,4,13,18],

$$\left| \omega^* = \frac{\omega}{\omega_D} = \frac{2\pi f}{\omega t^{z\gamma}} \right| \quad (5)$$

where ω_D is given by:

$$\left| \omega_D = \frac{k_B T}{3\pi \eta \xi^3} = \frac{k_B T_c}{3\pi \eta_0 \xi_0^3} t^{z\gamma} = \omega_0 t^{z\gamma} \right| \quad (6)$$

Here k_B is Boltzmann's constant, ξ is the correlation length and is η_0 the shear viscosity.

DATA ANALYSIS

The viscosity of binary liquid mixture methanol – cyclohexane is measured as a function of temperature using Brookfield viscometer. The critical temperature and critical concentration is determined.

The data of ultrasonic absorption coefficient were fitted using excel program by plotting $\frac{\alpha_c}{f^2}$ versus $f^{-1.06}$. Absorption coefficients at the critical concentration and any temperature data were taken at different frequencies [5 and 25MHz]. The relation of α_c versus T (°C) was plotted. The scaling function is plotted as a function of reduced frequency for the methanol – cyclohexane system.

RESULTS AND ANALYSIS

The critical concentration and critical temperature is obtained for the binary liquid mixture methanol – cyclohexane to be 30% by weight of methanol as critical concentration and 45.2°C as critical temperature. Shear viscosity is obtained to be 0.83cP as shown in Figure 1.

SPECIFIC HEAT CALCULATION

The specific heat at constant pressure at the critical temperature is calculated using the two scale fac-

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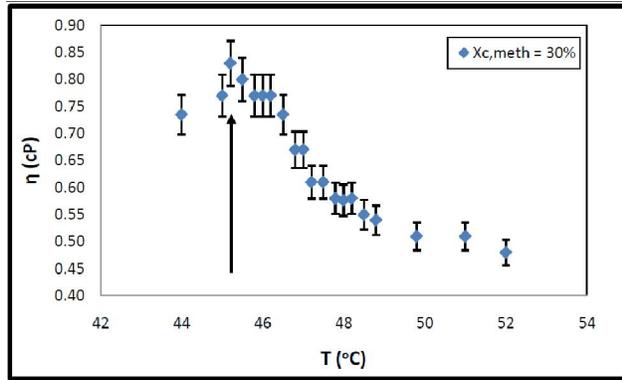


Figure 1 : The dynamic shear viscosity of methanol – cyclohexane as a function of temperature at concentration 30%by weight of methanol.

tor universality.

The universal constant R is given by:

$$R = \xi_0 \left\{ \frac{\bar{\alpha} \rho_c C_{pc}}{K_B} \right\}^{\frac{1}{3}} = 0.27 \quad (1)$$

Where: $\bar{\alpha} = 0.11$, $K_B = 1.3806 \times 10^{-23} \frac{J}{K}$ and $\xi_0 = 3.24 \times 10^{-10} m^7$

CALCULATION OF THE ADIABATIC COUPLING CONSTANT g

The adiabatic coupling constant for methanol – cyclohexane critical mixture is calculated by using the definition

$$g = \rho_c C_p \left(\frac{dT_c}{dP} - \frac{T \alpha_p}{\rho C_p} \right) \quad (2)$$

Using $\rho_c = 0.7695 \frac{gm}{cm^3}$, $C_p = 9.4239 \times 10^5 \frac{erg}{oC \cdot gm}$, $\frac{dT_c}{dP} = 3.4 \times 10^{-8} \frac{cm^2}{dyne} oC^{-1}$, $T =$

45.2°C and $\alpha_p = \rho \times 9.1 \times 10^{-3} oC^{-1}$

$g = -0.29$

ULTRASONIC ATTENUATION RESULTS

The binary liquid mixture methanol – cyclohexane has a critical temperature at 45.2°C. The absorption measurements were made for the frequencies 5 and 25MHz in the critical concentration sample

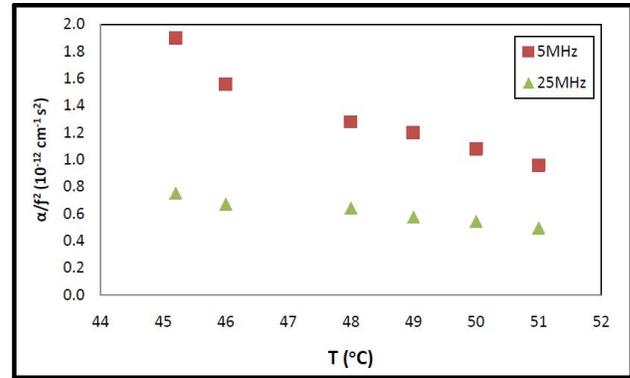


Figure 2 : The absorption coefficient $\frac{\alpha}{f^2}$ at frequencies 5 and 25MHz for the critical mixture methanol – cyclohexane as function of temperature.

of 30% by weight of methanol above the critical temperature

Ferrell and Bhattacharjee in the theory of the dynamic scaling proposed that the relation between α and f is fitted linearly for different values of frequency at the critical concentration and temperature. Values of the critical absorption coefficient at different frequencies from 5 – 45MHz

The slope from Figure 3 represents the critical part of the attenuation 1.8452 and the value of the intercept which represents the frequency – independent background term of is 5.5668 .

The magnitude of the slope from Figure 3 is expressed as:

$$\text{Slope (S)} = \left[\frac{\pi^2 C_{pc} g^2 u_c \bar{\alpha}}{2z \gamma T_c C_p^2(t_f)} \right] \left[\frac{a \omega_0}{2\pi} \right]^{\frac{a}{z\gamma}} = 1.8452 \times 10^{-7} cm^{-1} s^{0.94} \quad (3)$$

The characteristic temperature – dependent relaxation rate ω_0 is $3.5959 \times 10^{10} s^{-1}$.

The value of the dimensionless constant a is calculated from the equation:

$$a = \left(\frac{\omega}{\omega_0} \right) t_f^{-1.9} \quad (4)$$

The values of the dimensionless constant a from Figure 2 at the frequencies 5 and 25MHz are:

$$a (5MHz) = \frac{5 \times 10^6}{3.5959 \times 10^{10}} (0.1283)^{-1.9} = 6.8772 \times 10^{-3} \text{ and}$$

$$a (25MHz) = \frac{25 \times 10^6}{3.5959 \times 10^{10}} (0.1504)^{-1.9} = 5.0834 \times 10^{-3}$$

Sound velocity was measured by ultrasonic thickness gauge to be 1062. The values of the specific heat is calculated at the frequencies of 5 and

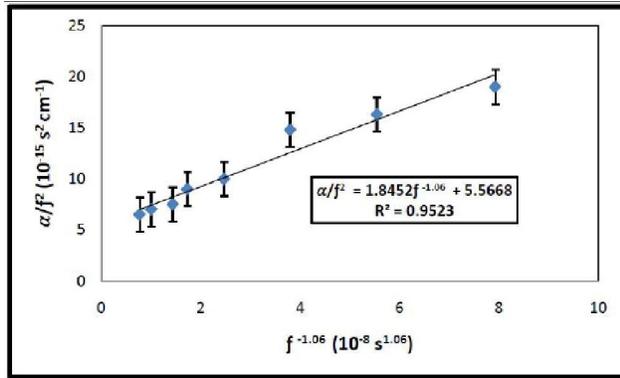


Figure 3 : The absorption coefficient $\frac{\alpha_c}{f^2}$ for critical mixture methanol – cyclohexane as function of $f^{-1.06}$.

25MHz using equation (3) as:

$$C_p(t_f)(5\text{MHz}) = 2817.07 \frac{\text{J}}{\text{Kg.deg}}$$

$$C_p(t_f)(25\text{MHz}) = 1698.01 \frac{\text{J}}{\text{Kg.deg}}$$

The dynamic scaling theory also described the scaling function $F(\omega^*)$ as a function of reduced frequency ω at various frequencies theoretically as:

$$F(\omega^*) = (1 + \omega^{*-0.5})^{-2} \quad (5)$$

The experimental data^[20] of the ratio of the absorption coefficient at the critical concentration and temperature $\alpha(x_c, T_c)$ to the absorption coefficient at the critical concentration and critical temperature $\alpha(x_c, T_c)$ are fitted along with the theoretical curve.

DIFFUSION COEFFICIENT CALCULATION

The diffusion coefficient D_o is calculated using the relation below:

$$\omega_0 = 2D_o \xi_0^{-2} \quad (6)$$

This equation was suggested by Kawasaki^[21] and Ferrell^[8] At the critical temperature D_o is calculated

$$\text{to be } 1.89 \times 10^{-5} \frac{\text{cm}^2}{\text{s}}$$

Discussion

Our experimental results of the dynamic viscos-

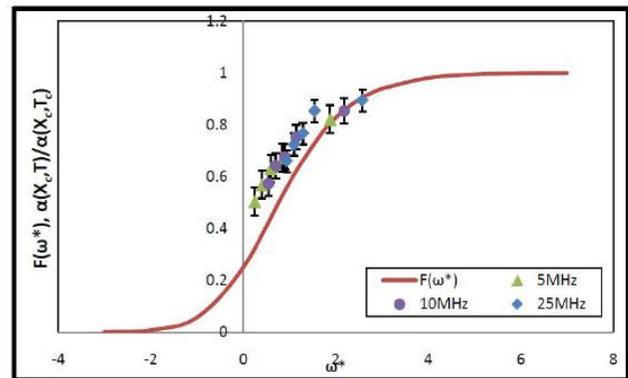


Figure 4 : The experimental values of $\frac{\alpha(x_c, T)}{\alpha(x_c, T_c)}$ along with the theoretical scaling function as a function of the reduced frequency.

ity showed an anomaly at the critical temperature and the critical concentration. The critical temperature is 45.2°C and the critical concentration is 30% by weight of methanol. The results showed a good agreement with Jacobs's experimental result, the critical concentration was found to be 29% by weight of methanol and $T_c = 45.1^\circ\text{C}$ ^[7]. The results were also in a good agreement with Behrends groups' measurements, the critical concentration was found to be 27.5% by weight of methanol while $T_c = 45.7^\circ\text{C}$ ^[16]. Our experimental results are also in a good agreement with Fast and Yun. Their values of critical concentration and critical temperature were found to be 28.9% by weight of methanol and 45.9°C respectively^[22].

The specific heat at the critical temperature and concentration of the critical mixture methanol – cyclohexane was calculated using the two scale factor

universality to be $C_{pc} = 94.24 \frac{\text{J}}{\text{K.Kg}}$ and it showed a good agreement with Kopelman teams' value of

$C_{pc} = 95.4 \frac{\text{J}}{\text{K.Kg}}$ ²³ The specific heat of the critical binary liquid mixture according to the dynamic scaling theory is a frequency dependent parameter and expresses the lagging in the internal degree of freedom and energy dissipation^[4].

The density of the critical mixture methanol – cyclohexane was measured to be $769.5 \frac{\text{kg}}{\text{m}^3}$.

TABLE 1 : Ultrasonic attenuation results in this work and previous studies.

The measured and calculated value	Our results	Other work
x_c^*	30%	29.0% ^(a) 27.5% ^(b) 28.9% ^(c) 45.1°C ^(a) 45.7°C ^(b) 45.9°C ^(c)
T_c^*	45.2°C	
C_{pc}^{**}	$94.24 \frac{J}{K.Kg}$	$95.4 \frac{J}{K.Kg}$ ^(d)
g^{**}	- 0.29(1 st method)	0.21 ^(b)
α_p^{**}	$7.00 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$	$1.28 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ ^(b)
α_{pc}^{**}	$3.797 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$	–
α_{pb}^{**}	$6.2401 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$	–
ρ_c^*	$0.7695 \frac{gm}{cm^3}$	–
η_o^*	0.83cP	–
D_o^{**}	$1.89 \times 10^{-5} \frac{cm^2}{s}$	–
u_c	$1062 \frac{m}{s}$	–

^(a): (Jacobs, 1986), ^(b): (Behrendset *al*, 2003), ^(c): (Fast and Yun, 1988), ^(d): (Kopelmanet *al*, 1984), *: measured value, **: calculated value.

The adiabatic coupling constant g for the critical mixture was to be - 0.29. The value was less than one which is in a good agreement with Behrends teams' value of g which is^[16]The difference between our value and Behrends value is due the difficulty in determination of the isobaric thermal expansion coefficient. Our value of the isobaric thermal expansion coefficient is $7.00 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ while Behrends value was $1.28 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ ^[16]. The difference between our value of the isobaric thermal expansion coefficient α_p and Behrends's value is may be of different values of critical concentration, critical temperature, noncritical dynamic shear viscosity and critical density. Since the adiabatic coupling constant gives a notation about the coupling between the critical density fluctuation and sound propagation^[4,3,18]. The g depends on pressure, temperature and isobaric thermal expansion coefficient.

The value of the adiabatic coupling constant g for alcohol – alkane systems is considered small

compared to the value of $g = 1.3$ for ethylammonium nitrate – n-octanol and $g = 2.1$ for isobutyric acid – water. These discrepancies are due to the difference in the values of the critical temperature, critical and background isobaric thermal expansion coefficients (α_{pc} , α_{pb}) and because of the difference in the magnitude of $\frac{dT_c}{dP}$ and α_p .

The negative sign of the value of g indicates that the phase separation near the critical point is induced by a sudden decrease in the pressure^[13].

The results of the absorption coefficients were plotted versus the temperature of two frequencies 5 and 25MHz. The temperature of the half of the maximum value of the absorption coefficients per the square of the frequency was determined for each frequency. The values were 51°C for 5MHz and 52°C for 25MHz and the reduced temperature for these values was calculated in order to calculate the dimensionless constant a . The values of a are 6.8772×10^{-3} at 5MHz and 5.0834 at 25MHz. The

reason that the data of the ultrasonic absorption coefficients at various frequencies were fitted with temperature but not concentration is that Bhattacharjee and Ferrell theory based on the temperature fluctuation resulting from the adiabatic compression and expansion of the mixture and so the critical behavior takes place only through the fluctuation in $\Delta T^{[4]}$.

The results of ultrasonic attenuation showed a linear behavior when $\frac{\alpha}{f^2}$ is plotted versus $f^{-1.06}$. The linear relation of the plotted data is in a good agreement with the dynamic scaling theory which predicts this behavior⁴. The slope of the plotted data of versus is 1.8452 and the intercept is 5.5668 represents the background absorption coefficient. The value rises monotonically as a function of the reduced frequency as it approaches to the critical point. The experimental data of as a function of the reduced frequency was compared to the theoretical relation $F(\omega^*) = (1 + \omega^{*-0.5})^{-2.4}$. It is found that the experimental results of are in a good agreement with the theoretical assumption by Ferrell and Bhattacharjee⁴. Compared to other theories, like the mode coupling theories, the experimental results of the dynamic scaling theory fit with the theoretical hypothesis better than mode coupling theories.

The characteristic temperature frequency ω_0 was calculated and it is found to be $3.5959 \times 10^{10} \text{ s}^{-1}$. The value of according to the dynamic scaling theory is calculated when the energy decay rate of a fluctuation is at $\xi^{-1.4}$.

The results of the different properties that have been calculated or measured for the binary liquid mixture methanol – cyclohexane compared with results from literature review are presented in TABLE 1.

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