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A STUDY OF VAN HOVE NON-FERMI SUPERCONDUCTORS AND DEGRADATION OF CRITICAL TEMPERATURE

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ABSTRACT

Using the model proposed by Anderson and Yin and Chakravarty, the non-Fermi behavior of copper oxide superconductors has been analyzed using an energy dependent density of states of the Van Hove (logarithmic) form. The critical temperature degradation due to non-magnetic impurities has been analyzed. The values of critical temperature decrease with non-Fermi parameter α. The width D of the Van Hove singularity affects critical temperature. The larger value of D gives large value of critical temperature.

Key words: High temperature superconductors, Non-Fermi liquid model, Van Hove density of states, Critical temperature degradation.

INTRODUCTION

High temperature copper oxide superconductors are materials with a normal state that cannot be described by Fermi liquid theory[1]. These are strong correlated electron materials. In one dimension, its behavior leads to the Tomonaga-Luttinger liquid[2,3]. Various scenarios were proposed to explain both; the superconducting and the normal state of the cuprates. Among these models, one is the non-Fermi liquid model. The model has a strong experimental support due to the many normal state properties that are not of the Fermi liquid type. The Tomonaga-Luttinger model, a non-Fermi liquid with no quasi-particles, is a one-dimensional model. The extension of the non-Fermi model to the higher dimensions is difficult and a complete microscopic theory in $d \geq 2$ does not exist. However, Balatsky[4] considered the superconducting state for $d = 2$ systems with a spectral density of the form $A(\Lambda k, \Lambda\omega) = \Lambda^{\alpha-1}(k, \omega)$ where α is the non-Fermi liquid parameter and depends on the strength of the interaction between electrons[5]. $\alpha = 0$ case correspond to the Fermi liquid model. Several properties of the Anderson non-Fermi liquid were discussed[6-13].

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Mathematical formulae used in the evaluation

In a non-Fermi liquid, the Green's function is written in the form[8].

$$G_0(\vec{k}, i\omega_n) = g(\alpha)\frac{\theta(\omega_n)e^{-i\frac{n\alpha}{2}} + \theta(-\omega_n)e^{-i\frac{n\alpha}{2}}}{\omega_c{}^{\alpha}(i\omega_n - \epsilon_k)^{1-\alpha}} \quad \ldots(1)$$

Where

$$g\,(\alpha\alpha = \frac{\pi\alpha}{2\sin(\frac{\pi\alpha}{2})} \quad \ldots(2)$$

ω_c a characteristic energy, cut off θ (x) the Heavisbide function, and α a parameter that describes the non-Fermi character of the liquid. The limit α → O corresponds to the standard Fermi liquid. The superconducting state is due to an attractive interaction V, and the superconducting critical temperature $\boldsymbol{T_c^0 \equiv T_c^0(\alpha)}$ is calculated using the Thouless criterion.

$$1 - \mathrm{VII}\ (0,0) = 0 \quad \ldots(3)$$

Where Π (0,0) is the particle-particle bubble given as –

$$\Pi(0,0) = T_c^0 \sum_n \int \frac{d^dk}{(2\boldsymbol{\pi})^d} G_0(\vec{k}, i\omega_n)\, G_0(-\vec{k}, -i\omega_n) \quad \ldots(4)$$

Where $\omega_n = 2\pi T_c{}^0(n + \frac{1}{2})$ are the Matsubara frequencies. Using Eq. (1), the particle-particle bubble becomes -

$$\Pi(0,0) = T_c{}^0 g^0(\alpha)\omega_c{}^{-2\alpha}\Sigma_n \int \frac{d^dk}{(2\pi)^d}\left[\frac{1.0}{(\varepsilon^2{}_k + \omega^2{}_n)^{1-\alpha}}\right] \quad \ldots(5)$$

In high-temperature superconductors, which are strong correlated systems, the density of states will be considered of the van Hove form[14,15].

$$N(\varepsilon) = N(0)\ln \in \left[\left|\frac{D}{\varepsilon}\right|\right] \quad \ldots(6)$$

Where D is another characteristic energy. It characterizes the width of the singularity. Taking the energy dependent density of states, the particle-particle bubble is now given by -

$$\Pi(0,0) = -2N(0)T_c^{\,0} g^0(\alpha)\omega_c^{-2\alpha}\Sigma_n \frac{1}{\omega_n^{\,2(1-\alpha)}}\int_0^\infty d\varepsilon \ln(\frac{\varepsilon}{D})\frac{1}{[1+(\frac{\varepsilon}{\omega_n})^2]^{1-\alpha}} \quad \ldots(7)$$

(Here, the upper intergration limit has been extended to infinity due to the rapid decrease of the integrand). The integral over energies has been calculated using the following formula -

$$\int_0^\infty dx \frac{\ln x}{[1+(bx)^2]^{1-\alpha}} = -\frac{1}{4b}B(\frac{1}{2},\frac{1}{2}-\alpha)[-\gamma - \ln 4 - \ln b - \psi(\frac{1}{2}-\alpha)] \quad \ldots(8)$$

Where B (x, y) is the beta function, γ the Euler's constant and ψ (x) the digamma Function.

(b = $\frac{\varepsilon}{\omega_n}$) Now with equation (8), Π (0, 0) becomes

$$\Pi(0,0) = -\frac{1}{2}N(0)T_c^{\,0} g^0(\alpha)\omega_c^{-2\alpha}B(\frac{1}{2},\frac{1}{2}-\alpha)\Sigma_n \frac{1}{\omega_n^{\,1-2\alpha}}[A(\alpha) - 2\ln(\frac{D}{\omega_n})] \quad \ldots(9)$$

Where $A(\alpha) = -\gamma - \ln 4 - \psi(\frac{1}{2}-\alpha)$ …(10)

The sum over Matsubara frequencies is rewritten as

$$\Sigma_n \frac{1}{\omega_n^{\,1-2\alpha}}[A(\alpha) - 2\ln(\frac{D}{\omega_n})] = \frac{1}{(2\pi T_c^{\,0})^{1-2\alpha}}[2S_1 A(\alpha T_c^{\,0}) + 4S_2] \quad \ldots(11)$$

With $A(\alpha T_c^{\,0}) = [A(\alpha) - 2\ln(\frac{D}{2\pi T_c^{\,0}})]$ …(12)

$$S_1 = \sum_{n=0}^{n_{\max}} \frac{1}{(n+\frac{1}{2})^{1-2\alpha}} \quad \ldots(13)$$

$$S_2 = \sum_{n=0}^{n_{\max}} \frac{\ln(n+\frac{1}{2})}{(n+\frac{1}{2})^{1-2\alpha}} \quad \ldots(14)$$

The value of $n_{\max}$ is given by $n_{\max} = \dfrac{D}{2\pi T_c^{\,0}} - \dfrac{1}{2}$

The sums are evaluated and the results are given by -

$$S_1 \approx \frac{2}{2^{2\alpha}} + \frac{1}{2\alpha}[(\frac{D}{2\pi T_c^{\,0}})^{2\alpha} - 1] \qquad \ldots(15)$$

$$S_2 \approx -\frac{\ln 4}{2^{2\alpha}} + (\frac{D}{2\pi T_c^{\,0}})^{2\alpha}[\frac{1}{2\alpha}\ln(\frac{D}{2\pi T_c^{\,0}}) - \frac{1}{(2\alpha)^2}] + \frac{1}{(2\alpha)^2} \qquad \ldots(16)$$

With these results, $\lambda = VN(0)$ the coupling constant and with $x = \dfrac{D}{2\pi T_c^{\,0}}$, the Thouless criterion (the equation for the critical temperature) becomes -

$$1 - \frac{\lambda}{\pi} g^2(\alpha)(\frac{D}{\omega_c})^{2\alpha} B(\frac{1}{2}, \frac{1}{2} - \alpha)\{(\frac{2}{2^{2\alpha}} - \frac{1}{2\alpha})x^{-2\alpha}\ln x[M(\alpha)(\frac{2}{2^{2\alpha}} - \frac{1}{4\alpha}) + \frac{\ln 4}{2^{2\alpha}} - \frac{1}{(2\alpha)^2}]x^{-2\alpha} + \frac{M(\alpha)}{4\alpha} + \frac{1}{(2\alpha)^2}\} = 0 \qquad \ldots(17)$$

Here $M(\alpha) = \psi(\frac{1}{2} - \alpha) - \psi(\frac{1}{2})$. In the $\alpha \to 0$ limit, equation (17) reduces to

$$\frac{1}{2}[\ln x]^2 + 2\ln x + \ln 4 - \frac{1}{\lambda} = 0 \qquad \ldots(18)$$

With the solution

$$T_c^{\,0}(\alpha = 0) = \frac{e^2}{2\pi} D \exp\{-\sqrt{4\ln(\frac{e}{2}) + \frac{2}{\lambda}}\} \qquad \ldots(19)$$

In the weak coupling limit gives the well-known formula for the Van-Hove superconductors.

In the $\omega \to 0$ limit, the inverse scattering time is logarithmic divergent. The infrared divergence can be removed introducing by regularization, a lower energy ω_r due to finite size of the scattering centers. Introducing an effective density of states -

$$N_{eff}(\alpha) = \frac{N(0)}{1+\alpha}\left(\frac{D}{\omega_c}\right)^{\alpha} \qquad ...(20)$$

The inverse scattering time is -

$$\frac{1}{\tau(\alpha)} = \pi n_i v^2{}_0 N_{eff}(\alpha)\ln\left(\frac{D}{\omega_r}\right) \qquad ...(21)$$

Here n_i is the impurities concentration, v_0 the strength of the short range impurities potential. The particle-particle bubble at finite temperature is given by -

$$\Pi(0,0) = -4N(0)T_c g^2(\alpha)\omega_c^{-2\alpha}\sum_{n=0}^{n_{\max}}\int_0^{\infty} d\varepsilon \ln\left(\frac{\varepsilon}{D}\right)\frac{(\varepsilon^2+\omega_n^2)^{\alpha}}{\varepsilon^2+\omega_n^2\left[1+\frac{A(\alpha)(\varepsilon^2+\omega^2{}_n)^{\frac{\alpha}{2}}}{\omega_n\tau(\alpha)}\right]^2} \qquad ...(22)$$

Using the Thouless criterion, the equation for the temperature degradation becomes -

$$1 - y^{2\alpha}\frac{T_2(\alpha)}{T_1(\alpha)} + y^{3\alpha}g(\alpha)d_c^{-\alpha}\cos\left(\frac{\pi\alpha}{2}\right)\frac{1}{\pi T_c^0\tau(\alpha)}\frac{T_3(\alpha)}{T_1(\alpha)}\frac{B(\frac{1}{2},\frac{3}{2}-\alpha)}{B(\frac{1}{2},\frac{1}{2}-\alpha)} = 0 \qquad ...(23)$$

Here $T_c^0 = T_c^0(\alpha)$ is the critical temperature for the clean case, and the following notations are used $y = \frac{T_c}{T_c^0}, d_c = \frac{\omega_c}{2\pi T_c^0}$ In α = 0, case equation (23) reduces to -

$$1 - \frac{2\ln c + \frac{1}{2}[\ln c]^2 + \ln 4}{2\ln b + \frac{1}{2}[\ln b]^2 + \ln 4} + \frac{1}{2\pi T_c^0(\alpha=0)\tau}\frac{5\ln c + 2\ln 4 + 4}{2\ln b\frac{1}{2}[\ln b]^2 + \ln 4} = 0 \qquad ...(24)$$

With $c = \frac{d_c D}{\omega_c y}, y = \frac{T_c(\alpha=0)}{T_c^0(\alpha=0)}, d_c = \frac{\omega_c}{2\pi T_c^0(\alpha=0)}, b = \frac{d_c D}{\omega_c}$ ...(25)

RESULTS AND DISCUSSION

We have analyzed the Anderson non-Fermi liquid model with a Von-Hove density of states. The critical temperature was calculated using the Yin and Chakravarty method. The non-Fermi character of the liquid affects drastically the values of the critical temperature that decreases with non–Fermi parameter α. The width D of the Von-Hove singularity affects the critical temperature. The large value of the D parameter gives larger value of the critical temperature, because of the enhanced density of states. The results are shown in Table 1. The scattering time in the presence of randomly distributed non-magnetic impurities was calculated. The ratio of the inverse scattering time R (in the non-Fermi case and when α = 0) are given in Table 2. The slope of R is affected by the width of the density of states singularity. The present work indicates that non-magnetic impurities affect the critical temperature of the high–temperature copper oxides superconductors[17-22]. The decrease of the critical temperature with disorder is different from the temperature degradation of Abrikosov-Gorkov model. The present model is a non–Fermi liquid model where non-magnetic impurities are assumed as pair breakers[23-25.]

Table 1: An evaluated results of $\frac{T_c^0}{D}$ as a function of α for λ = 1

α	$\frac{D}{\omega_c}$ **= 10**	$\frac{D}{\omega_c}$ **= 1**	$\frac{D}{\omega_c}$ **= 0.1**
0.0	0.22	0.20	0.21
0.05	0.20	0.19	0.17
0.10	0.18	0.16	0.14
0.15	0.15	0.14	0.11
0.20	0.13	0.12	0.08
0.25	0.10	0.10	0.06
0.30	0.08	0.08	0.04
0.35	0.05	0.05	0.03
0.40	0.03	0.02	0.01

Table 2: An evaluated results of ratio R [$\frac{\frac{1}{\tau(\alpha)}}{\frac{1}{\tau}}$] as a function of α for $\lambda = 1$

α	$\frac{D}{\omega_c}$ = 10	$\frac{D}{\omega_c}$ = 1	$\frac{D}{\omega_c}$ = 0.1
0.0	1.04	1.05	1.02
0.02	1.12	0.95	0.90
0.03	1.15	0.82	0.80
0.04	1.17	0.76	0.72
0.05	1.20	0.70	0.62
0.10	1.22	0.62	0.58
0.12	1.24	0.60	0.56
0.14	1.26	0.58	0.53
0.15	1.28	0.55	0.52
0.20	1.30	0.53	0.50

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