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A new wave energy generation system based on vibration theory

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ABSTRACT

Buoy-rope-sheave wave generation system is a new type floating and point absorbing wave energy generation system. Getting higher energy conversion efficiency and seeking the best damping coefficient are the important aspect of this paper. First, equation of buoy motion was established based on linear theory, harmonic vibration theory and viscous damping. The natural frequency and damping ratio was got from equation of motion. Then, the equation of motion was analyzed in frequency domain. The relation of damping coefficient and amplitude was got. Numerical calculation and optimization were taken for wave energy conversion using the relationship of energy dissipation in harmonic vibration. Finally, the result was verified by hydrodynamics software—AQWA, and the best damping coefficient was got. Results of numerical calculation and simulation basically consistent and damping coefficient was equal to $18000N \cdot s / m$. The above theoretical analysis and simulation can be used to different buoys in different sea conditions, and it provides theoretical references for designing and manufacturing buoy-rope-sheave wave generation system.

KEYWORDS

Wave energy; Forced vibration; Damping; Buoy-rope- sheave.



INTRODUCTION

Many countries attach great importance to the development and utilization of ocean wave energy. Many researchers have researched it for many years, and kinds of wave energy power generation device have been produced. The main forms of wave energy power generation device include oscillating water column type^{[1][2][3]}(such as the British LIMPET (land installed marine powered energy transformer), and the oscillating water column type Wave Power station designed by Guangzhou Institute of Energy Conversion, Chinese Academy of Sciences), oscillating buoy type^{[4][5]}(such as the UK Aqua buoy device, Power Buoy device), nodding duck type^[6](SALTER nodding duck Wave absorption Wave energy device), raft type^[7](such as McCabe Ocean Wave pump Wave Power device designed by Cork University and the Queen's University and Pelamis Wave energy device produced by Scotland Power Delivery company), pendulum type (Oyster pendulum Wave energy device developed by Scotland's green energy Corporation^[8]) and contraction channel type (350 kM stationary contraction channel device in Norway^[9] and Wave Dragon in Danish^[10]) devices, etc.

Buoy-rope-sheave wave generation system is a new point absorption wave power device, which has advantages of simple structure, low manufacturing and maintenance costs. Wave energy conversion efficiency will mainly be researched in this paper. In order to enhance the conversion efficiency and to seek the optimal damping coefficient, motion mathematical model of buoy in heave direction is established according to force analysis of buoy firstly. The heave motion of buoy is analyzed in frequency domain by mathematical model calculation and hydrodynamic simulation. Secondly, it is necessary to get the theoretical model of power generation based on motion mathematical model, and to analysis and optimize factors influencing the efficiency of power generation. Finally, the optimization result is verified using hydrodynamic simulation.

BUOY-ROPE-SHEAVE WAVE GENERATION SYSTEM

The sketch of buoy-rope-sheave wave generation system can be seen in figure 1. It includes buoy, generator, load, rope and gravity anchor. The rope links gravity anchor to generator shell sheave. Magnetic steel sleeve of generator is embedded on the sheave, which is the rotor. Rope sheave rotates with the rope to generate electricity when waves rise. Electrical energy is consumed by load. Rope will be automatically withdrawn with the action of coil spring when waves fell.

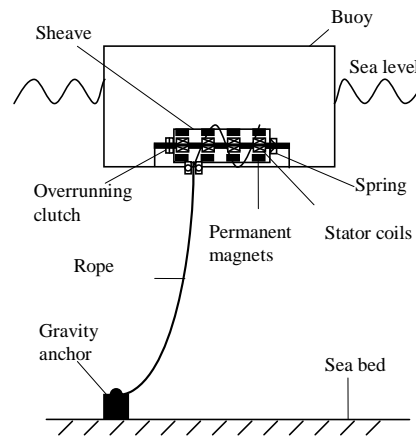


Figure 1 : The sketch of buoy-rope- sheave wave generation system

MOTION MATHEMATICAL MODEL OF BUOY

Buoy-rope-sheave wave generation system captures and converts wave energy to electrical energy in heave motion which is the key link of energy capture. It is key to studying heave motion of buoy with wave and energy capture. Motion mathematical model of buoy in heave direction is established firstly. Heave motion of buoy in waves is similar to forced vibration caused by bearing movement. Excited forces are the wave force and buoyancy. Damping is electromagnetic resistance of generator. Figure 2 shows force diagram of buoy, and vibration model can be seen in Figure 3.

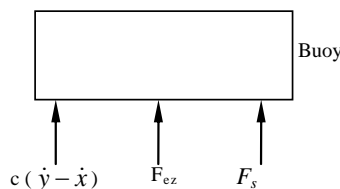


Figure 2 : Force diagram of buoy

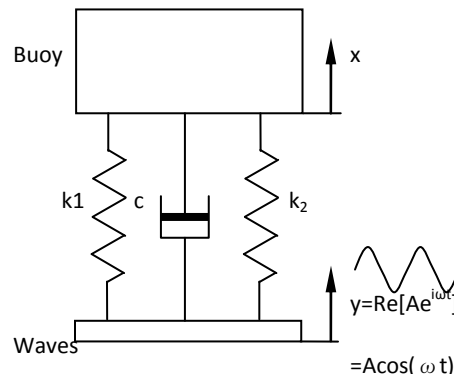


Figure 3 : Vibrational model of the device

Wave force

Wave force is a kind of fluid force caused by fluid dynamic pressure. Hydrodynamic pressure in flow field is $p = -\rho \frac{\partial \Phi}{\partial t}$, wave forces on the buoy can be obtained through integration for the buoy surface pressure^[11-13]. The expression of wave force in first order approximation is shown in eq.1.

$$F_{ez} = \iint_s p n ds = i\omega\rho \iint_s \phi n ds = \omega^2 \rho Y \iint_s \phi n ds \quad (1)$$

Hydrodynamic coefficient is defined by:

$$f = \omega^2 \rho \iint_s \phi n ds = e^{i\omega t} \omega^2 \mu + i\omega e^{i\omega t} \lambda \quad (2)$$

$$F_{ez} = Y e^{i\omega t} \omega^2 \mu + i\omega Y e^{i\omega t} \lambda = -\mu \ddot{x} - \lambda \dot{x} \quad (3)$$

where μ is added mass, and λ is added damping coefficient.

Added mass and added damping coefficient depend on buoy's shape and motion pattern. If frequency keeps invariant, added mass and added damping coefficient are constant^[14-17]. For different frequency, they are functions of frequency, and will be got in the frequency domain analysis.

Hydrostatic force

Phase difference between buoy and wave results in the hydrostatic force. Values of restoring force depend on the displacement difference between buoy and waves. It is shown in equation 4.

$$F_s = k(x - y) = k_1(x - y) + \rho g A(x - y) \quad (4)$$

where x is motion displacement of buoy, y is motion displacement of wave, k_1 is spring coefficient, $\rho g A$ is spring coefficient caused by buoyancy (A is water plane area of buoy).

Damping

Electromagnetic resistance of generator is the main damping. Permanent magnet generator is chosen, and its characteristics and damping force are given by following equations:

$$E = c_1 n \quad (5)$$

$$Fd\omega/2 = (E/R)^2 R \quad (6)$$

$$\omega = 2\dot{x}/d \quad (7)$$

$$n = 60\omega/2/\pi \quad (8)$$

Equation 9 is got by solving simultaneous equations (equation 5-8):

$$F = \frac{2c_1^2(30\omega)^2}{\pi^2 R d \omega} = \frac{3600c_1^2}{\pi^2 R d^2} \dot{x} = c\dot{x} \tag{9}$$

Where E is generator voltage, n is generator speed, ω is generator voltage angular velocity, R is resistance value of load, d is sheave diameter, F is damping force.

$$c = \frac{3600c_1^2}{\pi^2 R d^2} \tag{10}$$

Equation 10 can be obtained from Eq.9. Here, damping coefficient c depends on resistance value of load and sheave diameter which are constant, and damping and velocities are in opposite direction. So, it is viscous damping.

Mathematical modeling

Movement displacement expression of wave in the vertical direction is $y = \text{Re}[Ae^{i\omega t}] = A \cos(\omega t)$, and $x = \text{Re}[Xe^{i(\omega t - \varphi)}]$ represents motion displacement of buoy. According to Newton's Second Law, movement equations are given by following:

$$m\ddot{x} = F_{ez} + F_s + F_e \tag{11}$$

$$m\ddot{x} = -k(x - y) - c(\dot{x} - \dot{y}) - \mu\ddot{x} - \lambda\dot{x} \tag{12}$$

$$(m + \mu)\ddot{x} + (c + \lambda)\dot{x} + kx = c\dot{y} + ky \tag{13}$$

$c\dot{y} + ky$ respond excited force from damper and spring^[18-19]. Eq.14 shows the vibration standard form transformed from equation 13.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 2\zeta\omega_n\dot{y} + \omega_n^2y \tag{14}$$

Response vibration amplitude is shown as following:

$$X = \frac{A\sqrt{1 + (2\zeta\omega/\omega_n)^2}}{\sqrt{\left[1 - (\omega/\omega_n)^2\right]^2 + (2\zeta\omega/\omega_n)^2}} \tag{15}$$

where A is static displacement, and it is time-independent.

Assuming $H(\omega) = \frac{1}{1 - (\omega/\omega_n)^2 + i2\zeta(\omega/\omega_n)}$ (16)

Amplitude expression is as following:

$$X = A\sqrt{1 + (2\zeta\omega/\omega_n)^2} |H(\omega)| \tag{17}$$

Phase angle:

$$\varphi = \arctg \frac{2\zeta \left(\frac{\omega}{\omega_n}\right)^3}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + (2\zeta \frac{\omega}{\omega_n})^2} \quad (18)$$

Eq.19 shows the displacement response of buoy.

$$x = A \sqrt{1 + (2\zeta \frac{\omega}{\omega_n})^2} |H(\omega)| \cos(\omega t - \varphi) \quad (19)$$

$$\text{Natural frequency of buoy: } \omega_n = \sqrt{\frac{k}{m + \mu}}$$

$$\text{Damping ratio: } \zeta = \frac{c + \lambda}{2\sqrt{k(m + \mu)}}.$$

FREQUENCY DOMAIN ANALYSIS AND SIMULATION OF BUOY MOTION

Displacement response of buoy was analyzed in frequency domain to research the impact of excitation frequency on the steady-state response. When $\omega > 0$, amplification factor $|H(\omega)|$ can get the peak only on the condition of small damping ($\zeta < \frac{1}{\sqrt{2}}$). In this paper, it is need to discuss small damping. Amplitude frequency characteristic curves are got corresponding to $\zeta = 0.06, 0.1, 0.2, 0.3, 0.4, 0.5$, which is shown in Figure 4.

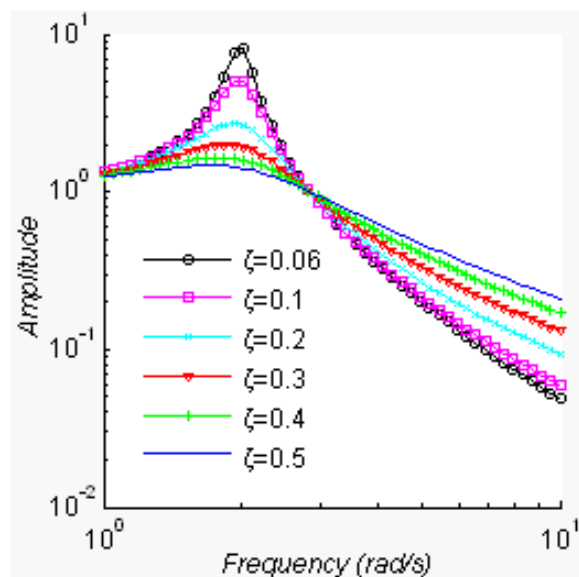


Figure 4 : Magnitude-frequency characteristic

From magnitude-frequency characteristic curve, it can be got that amplification factor $|H(\omega)|$ peaks when the frequency almost equals resonance frequency. Frequency response decreases with the increasing of damping ratio.

Frequency domain analysis of buoy displacement response carried out using the hydrodynamic analysis software-AQWA^[20]. The cylinder structure of buoy was chosen whose mass is 8800kg, diameter of bottom is 2.4m, and the depth of immersion is 1.9m. Figure 5 shows the power system in the given sea condition.

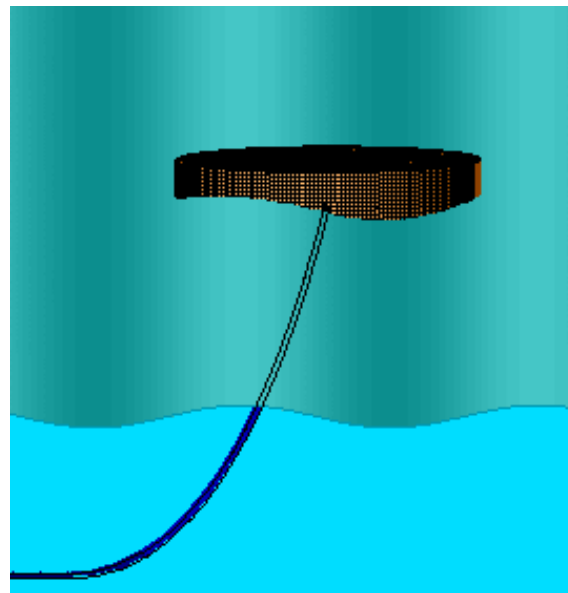


Figure 5 : Model of buoy

Figure 6(a) and 6(b) respectively show amplitude frequency characteristic curves which were got from hydrodynamic analysis software and mathematical model. Figures show that results of mathematical model and hydrodynamic analysis aligned fairly well.

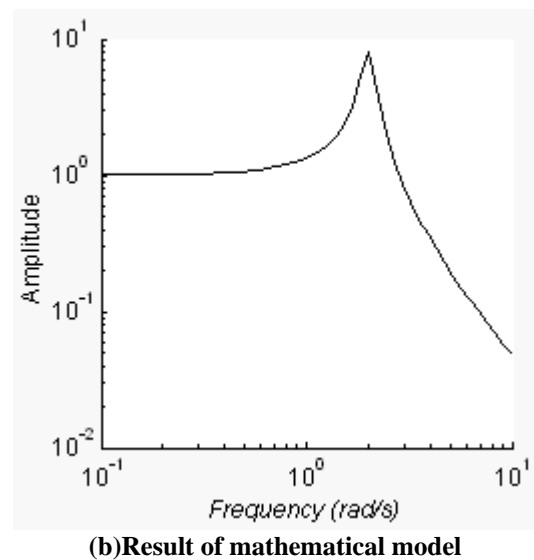
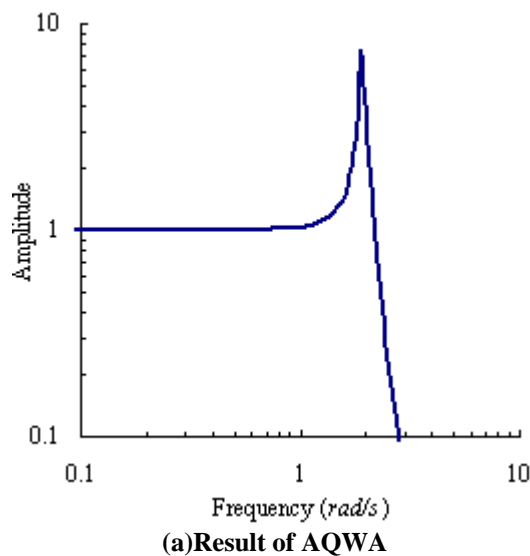


Figure 6 : Amplitude frequency curves

Additional mass and additional damping coefficient are functions of frequency. Laws of Additional quality and additional damping coefficient changing with the frequency were got in the frequency domain analysis, which is shown in Figure 7 and Figure 8.

The frequency analysis provided data of additional mass and additional damping coefficient. By fitting the analyzed data, the relationship of additional mass and additional damping coefficient with frequency was established with least square method. Fitting equations were derived as equation 17 and equation 18.

Fitting equation of additional mass:

$$\begin{aligned} \mu = & -6.6\omega^7 + 109.7\omega^6 - 706\omega^5 + 2148\omega^4 \\ & - 2917\omega^3 + 1059\omega^2 + 165\omega + 3960 \end{aligned} \tag{17}$$

Fitting equation of additional damping coefficient:

$$\begin{aligned} \lambda = & -3.5\omega^8 + 73\omega^7 - 608\omega^6 + 2572\omega^5 \\ & - 5674\omega^4 + 5823\omega^3 - 1852\omega^2 + 289\omega - 10 \end{aligned} \quad (18)$$

Fitted curves are presented in Figure 7 and Figure 8. It is obvious that added mass increases slightly with frequency at the beginning, then fell sharply. When the frequency is 2rad/s, it reaches the minimum value, and then it gradually rises. Additional damping coefficient of the initial value is 0, and increases with the frequency. The final value is also 0. When the frequency is 1.8rad/s, it reaches the maximum value.

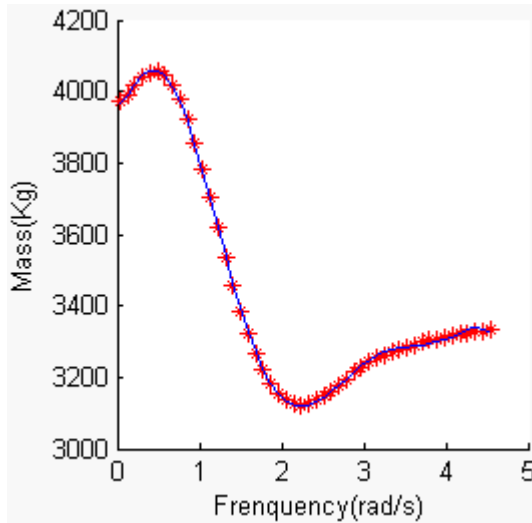


Figure 7 : Fitted curve of added mass

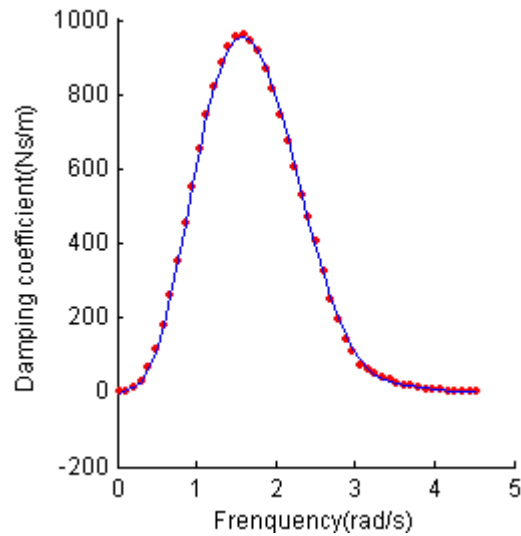


Figure 8 : Fitted curve of added damping coefficient

CALCULATION AND OPTIMIZATION OF CONVERSION EFFICIENCY

Conversion efficiency of wave energy is the key technology indicator of buoy-rope-sheave wave generation system. The energy absorbed by buoy from waves was transformed into electricity depending on the electromagnetic resistance of generator. Therefore, the electromagnetic resistance and buoy movement performance determine wave energy conversion efficiency of system eventually.

The work of electromagnetic resistance was calculated in the process of the buoy movement. It is the input power of generator. Eq.19 shows work of electromagnetic resistance over a period of time:

$$W = \int c \dot{x} dx = \int_0^{2\pi/\omega} c \omega^2 X^2 \sin^2(\omega t - \varphi) dt = c \pi \omega X^2 \quad (19)$$

The input power of generator is proportional to damping coefficient c , excitation frequency ω and amplitude X , but is independent of natural frequency. From amplitude frequency curves, it can be got that amplitudes decrease when damping coefficients increase. The total energy of buoy absorbed is as following:

$$W_T = (c + \lambda) \pi \omega X^2 \quad (20)$$

$$X = \frac{\sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} \quad (21)$$

$$\omega_n = \sqrt{k/(m + \mu)} \quad (22)$$

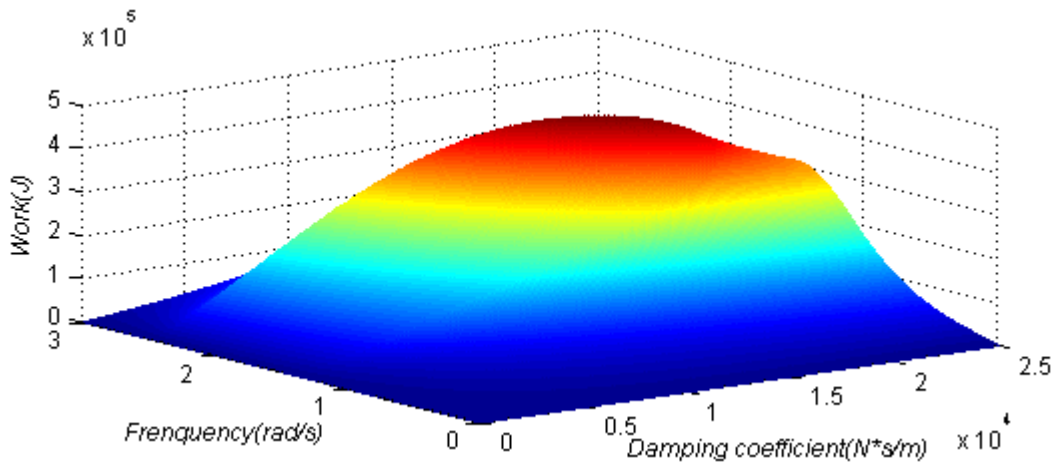


Figure 9 : The relationship of wave energy conversion power and frequency and damping coefficient

Maximum average power appeared when $c = 18610 N \cdot s / m$ and $\omega = 1.98 rad / s$, which is just at the resonant frequency.

Figure 9 shows calculated results.

Results of calculation are verified by the hydrodynamic analysis software-AQWA. Displacement response of buoy was analyzed in time domain to get the power of damping resistance. Based on frequency domain analysis, amplitude frequency response is a maximum when $\omega_n = \sqrt{k / (m + \mu)}$. So, the displacement experiment of buoy was simulated with different damping coefficients, assuming frequency of wave is 2rad/s, and wave height is 0.4m for test. The work of damping force is as following:

$$dW = dPdt = cv^2 dt$$

As long as getting the volocities, power of generator for some time can be got and compared. Values of damping coefficient are respectively $12000 N \cdot s / m, 15000 N \cdot s / m, 18610 N \cdot s / m, 23000 N \cdot s / m, 30000 N \cdot s / m$ and $40000 N \cdot s / m$. Figure 10 shows velocity curves of buoy with different damping coefficients, which indicates velocity decreases with damping coefficient increasing.

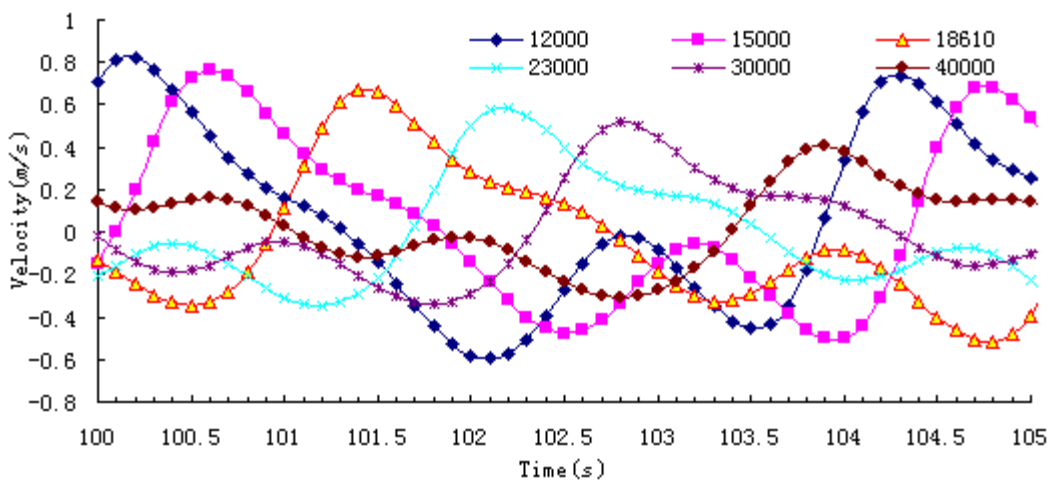


Figure 10 : Velocity curves of different damping coefficients

Power of generator was got according to velocities and $dW = cv^2 dt$, and is shown in Figure 11. The work of electromagnetic resistance rises first, and then declines, which reaches a maximum when $c = 18000 N \cdot s / m$. At this point, wave energy conversion efficiency is optimal, that is in accordance with the result of mathematical model.

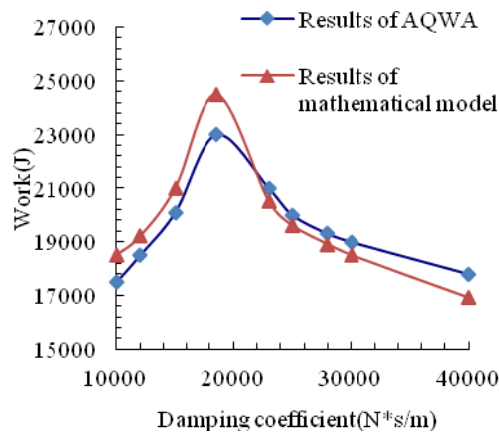


Figure 11 : Wave energy conversion power curve with different damping coefficients

CONCLUSIONS

1. Equation of buoy motion in heave direction is established based on linear theory, harmonic vibration theory and viscous damping. The mathematical model is analyzed in frequency domain.

2. The mathematical model is validated correct using the simulated analysis based on hydrodynamic methods. In the meantime, it provided data of additional mass and additional damping coefficient. By fitting the analyzed data, the relationship of additional mass and additional damping coefficient with frequency was established with least square method.

3. By calculating the actual mathematical model of one buoy, main factors influencing the power generator are analyzed and calculated. Optimal damping coefficient was got, and experimentally validated, which is $18000 N \cdot s / m$. This way of analysis can be used in design process of buoy-rope-sheave wave generation system with different buoys and different sea condition.

4. Theoretical analysis, calculated results and hydrodynamic simulation provide a base for research and development of buoy-rope-sheave wave generation system.

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