



## Full Paper

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Received: January 15, 2014  
Accepted: April 12, 2014

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# A model of violation and superviolation of Bell's inequality in local quantum mechanics

## Abstract

A model for violation and even superviolation of the Bell's inequalities in coincidence experiments with photons in local \qm\ is presented. The model is based on assumption that time retardation or losses in an analyzer depend on angle between linear polarization of an incident photon and the analyzer's axis.

Entangled states of separated particles, introduced in<sup>[1,2]</sup> make quantum mechanics nonlocal. This nonlocality is manifested by violation of Bell's inequalities certified in many experiments, where correlation of particle's polarizations in coincidence detection were measured. However below, it will be shown that violation of the Bell's inequality in coincidence experiments can be obtained even

in the local quantum mechanics.

Let's consider an experiment, which scheme is shown in Figure 1

The source  $S$  radiates photons assumed to have individual polarizations along some vector  $c$ , which has uniform angular distribution in the plane orthogonal to propagation direction. It is the common belief that the Bell's

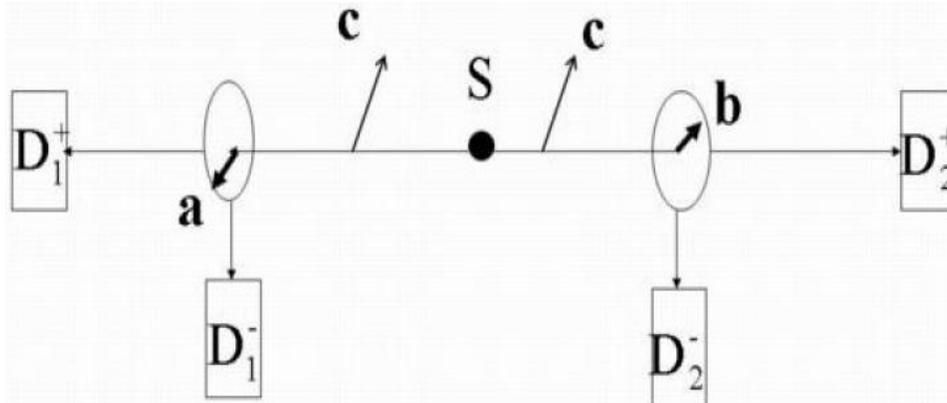


Figure 1 : Scheme of the experiment on coincident measurement of a correlation of polarizations of two photons radiated by a source  $S$ . The source radiates two photons with parallel polarizations  $c$ , which has a uniform angular distribution around direction of the photons flight paths. Polarizing beam splitters with axes  $a$  and  $b$  transmit photons along one of two channels toward the detectors  $D_{1,2}^{\pm}$

inequalities are not violated in such a case. However, it is shown below, that they can be violated, which supports the results of the numerical experiment reported in<sup>[3]</sup>. Moreover, one can predict in some experiments even superviolation, where correlation coefficient surpasses the maximal value  $2\sqrt{2}$ .

To be specific let's look at the most popular inequality<sup>[4]</sup>

$$-2 \leq S \leq 2 \tag{1}$$

Where

$$S = E(a,b) - E(a,b') + E(a',b') + E(a',b), \tag{2}$$

and  $E(a,b)$  is a correlation of polarizations of two particles registered after two analyzers with their axes along unit vectors  $a$  and  $b$  in an experiment depicted in Figure 1.

The definition of correlation function is the most important part of this letter. Usual definitions involve some predetermined classical functions<sup>[5]</sup> and does not address the specific features of coincidence experiments such as arrival time of particles and time window, which means that the width  $w$  of the time window is large enough or  $w = \infty$ , and no other particle can enter any of the detectors inside this window.

We suppose that the radiated photons with their individual polarizations interact with analyzers  $a$  and  $b$  quantum mechanically, i.e. probability of a photon with polarization  $c$  to be transmitted through an analyzer with its axis  $a$  is equal to  $P_+(a) = (a \cdot c)^2 = \cos^2(\alpha - \xi)$ , where  $\alpha, \xi$  are azimuthal angles of vectors  $a$  and  $c$  defined with respect to some axis normal to the propagation direction. In the following this axis will be chosen along the vector  $a$ , so  $\alpha = 0$ . The angle  $\xi$  will be assumed to have uniform distribution  $d\xi/2\pi$ . Thus the correlation of registrations looks as in<sup>[3]</sup>

$$E(a,b) \equiv E(\beta) = \frac{P_{++}(a,b) + P_{--}(a,b) - P_{+-}(a,b) - P_{-+}(a,b)}{P_{++}(a,b) + P_{--}(a,b) + P_{+-}(a,b) + P_{-+}(a,b)}, \tag{3}$$

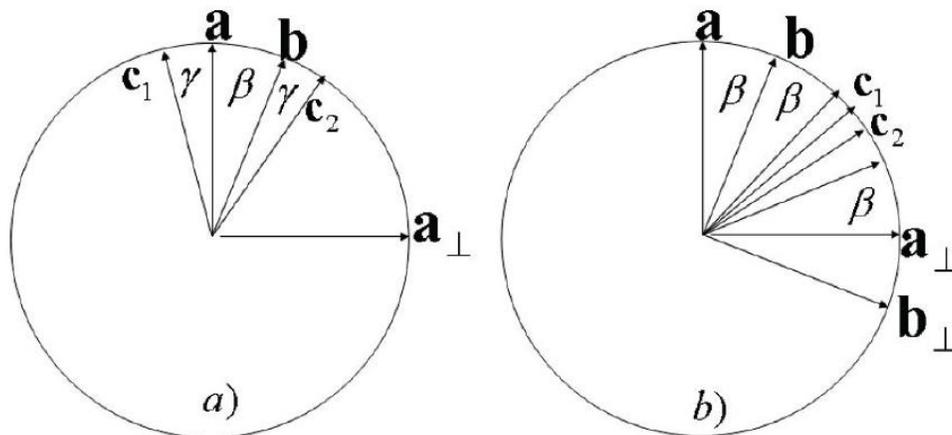


Figure 2 : Restriction of integration interval  $[c_1, c_2]$  in the case  $\beta = \beta_0$ . Coincidence takes place only if the angular distance of the photon polarization  $c$  from one of the analyzer's axes is not larger than  $\beta + \gamma$  for some fixed  $\gamma$ . a) Calculation of probability of the type  $P_{++}(\beta = \beta_0), P_{--}(\beta = \beta_0)$  and also  $P_{+-}(\beta = 3\beta_0), P_{-+}(\beta = 3\beta_0)$ . b) Calculation of probability of the type  $P_{+-}(\beta = \beta_0), P_{-+}(\beta = \beta_0)$  and also  $P_{++}(\beta = 3\beta_0), P_{--}(\beta = 3\beta_0)$ .

where, say,  $P_{\pm}(a,b)$  is the probability of registration by detectors  $D_{1,2}^{\pm}$  in coincidence, and  $\beta$  is the angle between vectors  $a$  and  $b$ .

The analyzers are supposed here to be without losses, and efficiency of registration after analyzers is supposed to be the same for all the detectors. Because of definition Eq. (3) this efficiency can be put to unity.

The probabilities in Eq. (3) can be calculated analytically. For instance,

$$P_{++}(a,b) \equiv P_{++}(\beta) = \int \frac{d\xi}{2\pi} \cos^2(\xi) \cos^2(\beta - \xi) \Theta(|t_1 - t_2| < w), \tag{4}$$

where  $w$  is the width of the coincidence window,  $t_{1,2}$  are the time delays of the moment of registration and  $\Theta$  is the step function equal to unity, when inequality in its argument is satisfied, and to zero otherwise.

The goal is to calculate all these probabilities and to show for some particular case,  $\beta = \beta_0 = \pi/8$ , that the inequality

$$S = 3E(\beta) - E(3\beta) < 2 \tag{5}$$

can be violated notwithstanding that the photons are not entangled.

In the following, like in<sup>[3]</sup>, it is supposed that the time difference  $\Delta t = |t_1 - t_2|$  depends on angular distance between vector of photons polarization  $c$  and axes of analyzers. For instance, one can suggest that coincidence counting stops, when this angular distance is larger than  $\beta + \gamma$  for some fixed parameter  $\gamma$ . It means that Eq. (4) can be represented as

$$P(\beta) = \int_{\xi_1}^{\xi_2} \frac{d\xi}{2\pi} \cos^2(\xi) \cos^2(\beta - \xi), \tag{6}$$

where  $\xi_{1,2}$  correspond to limiting positions  $c_{1,2}$  of photon polarizations in Figure 2.

For  $P_{++}(\beta)$  integration interval is  $-\gamma < \xi < \beta + \gamma$ , as is shown in Figure 2a), and for  $P_{+-}(\beta)$  integration interval is  $2\beta - \gamma < \xi < \beta + \gamma$ , as is shown in Figure 2b). If  $P_{++}(\beta) =$

$P_-(\beta)$  for  $\beta = \beta_0$  and a given  $\gamma$  is denoted as  $A(\gamma)$ , and  $P_+(\beta) = P_{\cdot+}(\beta)$  is denoted as  $B(\gamma)$ , then correlation Eq. (3) takes the form

$$E(\beta_0, \gamma) = \frac{A(\gamma) - B(\gamma)}{A(\gamma) + B(\gamma)}, \tag{7}$$

and the quantity Eq. (5), as is easy to check, becomes

$$S(\beta_0, \gamma) = 3E(\beta_0, \gamma) - E(3\beta_0, \gamma) = 4 \frac{A(\gamma) - B(\gamma)}{A(\gamma) + B(\gamma)}. \tag{8}$$

It is understandable that in the limiting case, when  $\gamma < \beta_0/2$ , the integration interval in Figure 2b) shrinks to zero, therefore  $B(\gamma \leq \beta_0/2)$  becomes zero too, and  $S(\beta_0, \gamma)$  in (8) becomes 4, which is larger than maximal possible value for entangled states  $S_{\max} = 2\sqrt{2} = 2.82$ . So in this case we have superviolation of the Bell's inequality.

Dependence of the function  $S(\beta_0, \gamma)$  on  $\gamma$ , where the first variable is omitted, is shown in Figure 3. It is seen from there that  $S(\gamma) > 2$  up to  $\gamma = 0.6035 \approx 1.6\beta_0$ .

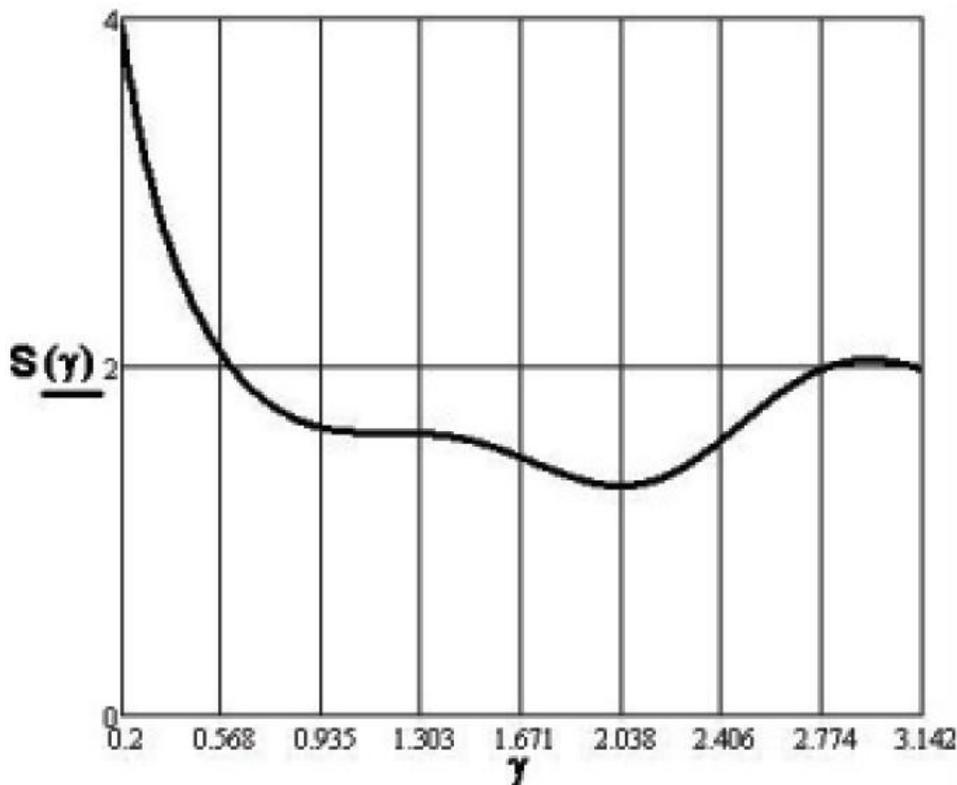


Figure 3 : Dependence of the function  $S(\beta_0, \gamma)$  on  $\gamma$  (the first variable is omitted).

For conclusion it is necessary to tell that all coincidence experiments devoted to verification of violation of Bell's inequalities have a preparation stage, when coincidence is tuned.

This preparation stage is never reported. However this stage can be crucial for "success" of the experiments, as follows from the above considerations.

### ACKNOWLEDGEMENT

I am to mention here with gratitude the arXiv admin Dr. Jake Weiskoff. His opinion that my articles on EPR paradox are not much related to quantum physics, but have a wider significance for all the general physics is appreciated.

### REFERENCES

- [1] A. Einstein, B. Podolsky, N. Rosen; Phys. Rev., **47**, 777 (1935).
- [2] D. Bohm, Y. Aharonov; Discussion of experimental proof for the paradox Einstein, Rosen, and Podolsky, Phys. Rev., **108**, 1070 (1957).
- [3] Hans De Raedt, Koen De Raedt, Kristel Michiels, Koenraad Keimpema, Seiji Miyashita; Event-based computer simulation model of Aspect-type experiments strictly satisfying Einstein's locality conditions, J. Phys. Soc. Jpn., **76**, 104005 (2007).
- [4] J.F. Clauser, M.A. Horn, A. Shimony, R.A. Holt; Proposed experiment to test local hidden-variable theories, Phys. Rev. Lett., **23**, 880 (1969).
- [5] John S. Bell; On the EPR paradox, Physics, **1**, 195-200 (1964).