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A fleet size and mix vehicle routing problem with quantity discounts for outsourcing

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ABSTRACT

In view of the fleet size and mix vehicle routing problem with quantity discounts for outsourcing, a non-linear programming model is established; and an evolutionary algorithm based on the shortest path encoding approach is designed to solve the model by minimize the variable costs and outsourcing costs. Through experimental analysis, it is shown that: the mutation probability and population size affects the optimal fitness defined by total costs; the higher mutation probability and bigger population size would quicken the convergence speed and quality of the evolutionary algorithm; the bigger population size would cost more computational time; other parameters impose no significant effect on experimental results. Based on these results, optimal parameters for the algorithm are determined. By a case, the performance and the effects of the algorithm are demonstrated.

KEYWORDS

Fleet size and mix vehicle routing problem; Evolutionary algorithm; Shortest path; Outsourcing; Quantity discounts.

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INTRODUCTION

This paper studies the mixed vehicle routing optimization problem with quantity discounts in background of logistical service outsourcing. Reasonable distribution approaches can reduce the logistical cost. Quantity discount strategy can attract customers and gain scale benefit for the fleet themselves. Moreover, in the market of fierce competition, coordination and double-win strategies are effective means for long-term development of logistical companies. The problems in fleet operations management usually relate to vehicle routing problem (VRP) or its variants. VRP means that the distribution center distributes the goods to a certain number of customers with demands. A fleet is responsive for distributing the goods with proper driving routes organized to meet the customer demand under certain constraints and the minimization of cost and time. Heterogeneous fleet vehicle routing problem (HFVRP) is an extension of the VRP, which determines the configuration of the vehicle types and the quantity of each type to help the fleet complete the distribution with minimal cost. HFVRP is widely studied by scholars, e.g. Belfiore and Yoshizaki[1], Brandão [2], Choi and Tcha[5], Prins[7] and Subramanian et al.[12]. When the variable cost of the vehicle is independent with its type and the quantity of available vehicles of each type is infinite, it is called fleet size and mix vehicle routing problems (FSMVRP). FSMVRP is also a hot problem studied by many scholars, e.g. Brandão[2], Bräysy et al.[4], Liu et al.[6], Renaud and Boctor[8] and Repoussis and Tarantilis[9]. Liu et al.[6] pointed out that FSMVRP is a NP-hard problem. Liu et al.[6] designed a hybrid genetic algorithm based on shortest path decoding for the FSMVRP. Renaud and Boctor[8] designed a heuristic algorithm based on sweep algorithm. Brandão[2] use the Tabu search algorithm to solve multi-type vehicle routing problem and mixed vehicle routing problem respectively. Other scholars (e.g. Liu et al.[6] and Subramanian et al.[12] combine the Heuristic algorithm with other intelligent algorithm such as genetic algorithm to solve the problem.

In the background of logistical outsourcing, this paper studies the FSMVRP with following new features, referred to as FSMVRPQD. Firstly, different vehicles have different capacities and costs. Outsourced vehicle has fixed cost, namely outsourcing cost. Variable costs include fuel and human resources cost, and it is assumed that the variable cost is proportional to the driving distance. Secondly, mixed vehicle routing problem with quantity discounts is considered, which takes the constraints like time window, capacity limitation into account. Thirdly, based on the shortest path decoding proposed by Liu et al.[6] for hybrid genetic algorithm, this paper designs solution method by modifying fitness function and adding time window in the decoding process, according to characteristics of the time window and the quantity discount.

PROBLEM

$N = \{0,1,2\cdots,n,n+1\}$ is a node set, including 1 depot and n customers, among which 0 and the hypothetical node n+1 represent the depot, others nodes are customers. For customer i ($i \in N \setminus \{0,n+1\}$), the demand is UL_i; service time at the customer is S_i; and the servicing time-window is $[U_i, L_i]$. The traveling distance between any two nodes $i \in N$ and $j \in N$ is $D_{i,j}$. The types of the vehicles of the fleet are denoted as set $TR=\{1\ ;\cdots, NT\}$. The quantity of the vehicles for each type is presumed to be infinite. The vehicular capacity, average speed and unit fixed costs of the vehicles of type $k \in TR$ are $MC_{k\in TR}$, $V_{k\in TR}$ and $RC_{k\in TR}$ respectively. $T_{i,j}^{k}$ is the transportation time for the vehicles of type $k \in TR$ travel from node $i \in N$ to node. TC is the transportation cost of traveling a unit of distance. Aimed at minimizing the total costs f (the sum of variable cost vc and fixed cost fc) of completing N tasks, the actual number of vehicles of each type and the route of each vehicle should be solved. The outsourcing discount of vehicular type $k \in TR$ is θ_k, which relates to the number of the outsourced vehicles at this time, denoted as a function, $\theta_k(m_k)$. Another point that needs to explain is that the total cost includes variable cost and fixed cost. The former is undertaken directly by the logistical outsourcing companies who complete the distribution task, while the latter (i.e. outsourcing cost) indicates the transportation cost paid to logistical companies by the customer. This paper considers the benefits of logistical companies and customers, which facilitate the cooperation of the companies in supply chains. A double-win situation will be achieved for the transportation companies and the customers. Moreover, reasonable practical conditions can be considered as follows: 1) The capacity and fixed cost of different types of vehicles are different, the larger the capacity, the more the cost; 2) Only one vehicle can be allowed to be responsible for the distribution of each customer, that is, customer requirements can not be split; 3) The distribution quantity of any customer is less than the biggest vehicular capacity; 4) Transportation companies outsource the distribution tasks to other special transportation company, which means that to satisfy the logistical demand by renting distribution capacity. The average outsourcing cost of a vehicle depends on the quality of the outsourced vehicles. The more the outsourced vehicles indicates the lower the discount.

MODEL

A nonlinear programming model is established for the above problem. Four groups of decision variables are defined as follows. First, x_i^k is the time when the vehicle of type $k \in TR$ arrives at the customer $i \in N \setminus \{0, n+1\}$. Second, m_k is the used number of vehicles of type $k \in TR$. Third, $y_{i,j}^k \in \{0,1\}$. $y_{i,j}^k = 1$, if the vehicle of type $k \in TR$ serves j after serving i; or else, 0. Fourth, z_i^k represents the unloaded quantity when the vehicle of type $k \in TR$ leaves customer i.

Minimize: **(1)**

$$f = vc + fc$$

s.t.

$$fc = \sum_{k \in TR} m_k \cdot RC_k \cdot \theta_k(m_k) \quad \textbf{(2)}$$

$$vc = \sum_{i,j \in N, k \in TR} y_{i,j}^k \cdot D_{i,j} \cdot TC \quad \textbf{(3)}$$

$$T_{i,j}^k = \frac{D_{i,j}}{V_k}, \forall i, j \in N, k \in TR \quad \textbf{(4)}$$

$$\sum_{j \in N \setminus \{0, n+1\}} y_{0,j}^k = m_k, \forall k \in TR \quad \textbf{(5)}$$

$$\sum_{i \in N \setminus \{0, n+1\}} y_{i,n+1}^k = m_k, \forall k \in TR \quad \textbf{(6)}$$

$$x_j^k \geq T_{0,j}^k - MT(1 - y_{0,j}^k), \forall j \in N \setminus \{0, n+1\} \quad \textbf{(7)}$$

$$\forall i, j \in N \setminus \{0, n+1\}:$$
$$x_j^k \geq x_i^k + S_i + T_{i,j}^k - MT(1 - y_{i,j}^k) \quad \textbf{(8)}$$

$$\forall k \in TR,\ j \in N \setminus \{0, n+1\}: \quad z_j^k \geq z_i^k + UL_j - MC_k (1 - y_{i,j}^k) \tag{9}$$

$$\sum_{i \in N \setminus \{n+1\}, k \in TR} y_{i,j}^k = 1, \forall j \in N \setminus \{0, n+1\} \tag{10}$$

$$\sum_{j \in N \setminus \{0\}, k \in TR} y_{i,j}^k = 1, \forall i \in N \setminus \{0, n+1\} \tag{11}$$

$$\forall j \in N \setminus \{0, n+1\}: \quad \sum_{i \in N \setminus \{0,n+1\}, k \in TR} y_{i,j}^k = \sum_{i \in N \setminus \{0,n+1\}, k \in TR} y_{j,i}^k \tag{12}$$

$$L_i \leq x_i^k \leq U_i, \forall i \in N \setminus \{0, n+1\}, k \in TR \tag{13}$$

$$z_i^k \leq MC_k, \forall i \in N \setminus \{0, n+1\}, k \in TR \tag{14}$$

$$\theta_k(m_k) = \begin{cases} \lambda_{k,1} = 1,\ 0 < m_k \leq \Omega_{k,1} \\ \lambda_{k,2}, \Omega_{k,1} < m_k \leq \Omega_{k,2} \\ \lambda_{k,3}, \Omega_{k,2} < m_k \leq \Omega_{k,3} \\ \vdots \end{cases}, \ \lambda_{k,1} = 1 > \lambda_{k,2} > \lambda_{k,3} \cdots \tag{15}$$

The model is defined through Equations (1)-(15). The objective function (1) minimizes the total cost including fixed cost and variable cost. Constraint (2) calculates the fixed cost, where $RC_k(m_k)$ is the average cost of outsourcing vehicle of type k for a unit time. Constraint (5) calculates the variable cost. Constraint (6) restricts the times for vehicles of type k returning to the depot. Obviously, the leaving times of the vehicle equals to the returning times. In other words, each vehicle that starts from the depot should go back to the depot. Constraints (7)-(9) connect x and y, and z and y to ensure the access sequence of the customers. Constraints (10)-(12) are flow constraints. The entering and leaving times of each customer point are at most 1 times. That is, each customer point can only be served for one time. Constraints (13) mean that each customer is served within the time window. Constraint (14) ensures that the vehicular capacity should not be exceeded. Constraint (15) defines the discount of the vehicles of each type, where λ ($0 < \lambda \leq 1$) is the discount, Ω is the parameter to specify different discounts for different ranges of outsourced vehicles. Constraint (15) implies that the more outsourced vehicles, the less discount and the less fixed costs for outsourcing a single vehicle.

EVOLUTIONARY ALGORITHM

Algorithm 1 is proposed below to solve the nonlinear programming model established in Section 3. It is an evolutionary algorithm whose fitness is the sum of the fixed costs and variable costs. Shortest path algorithm is used for decoding the chromosome in Algorithm 1. The detailed decoding process is shown in Algorithm 2.

Algorithm 1 (Evolutionary algorithm for FSMVRPQD)

Input: population size P_s, crossover probability P_x, mutation probability P_m, iterations P_g.
Output: Optimal solution of distribution.

Process:

Step 1. Initialization. Form a population P_o by random generating M permutations of *1, 2, …, N* as chromosomes.
Step 2. Evaluation: Decoding and calculate fitness.
Step 3. Reproduction
1) Selection. Form paring set with rank-based selection approach.
2) Crossover. Form a new population P_n by using crossover operator based on PMX and controlling crossover probability with parameter m_x.
3) Mutation. Update the new population P_n by using two point replacement mutation operators and controlling mutation probability with parameter m_m.
Step 4. Evaluation: Calculate fitness of individuals after decoding the chromosomes in population P. According to the fitness update the elite.
Step 5. Termination conditions. If termination conditions are not met, then turn to Step 3; Or else, return the elite.
(1) Chromosome encoding and decoding

Chromosome is encoded by customer sequence. For the customer set $\{1,\cdots,n\}$, the genes in a chromosome is presented by $Ch=\{g(1),g(2),g(3),\cdots,g(n)\}$, where each element is corresponding to a customer numbers. In the decoding process of a chromosome, the routes and the assessing orders of the customers should be determined. Liu et al.[6] proposed a decoding method based on shortest path algorithm for the mixed vehicle and time window conditions. For a chromosome $Ch=\{g(1),g(2),g(3),\cdots,g(n)\}$, the customers are divided into several groups according to customer sequence represented by the chromosome. Each customer group is distributed by a vehicle with conditions: the total demand of each group must not exceed the vehicular capacity of the selected vehicular type; the service time (corresponding to the arriving time of the vehicle to the customer) of each customer in each group must meet the time window defined for the customer. Moreover, the solution corresponding to the chromosome should be the solution with minimal total variable costs among the feasible solutions which meet the above two conditions. Variable cost is measured by the total traveling distance of the routes.

First, a directed acyclic graph is formed according to the chromosome through the following ways. It is presumed that the indices of the vehicular types are in ascending order of the capacity of the vehicles, denoted as $MC_k < MC_{k+1}, k=1,\cdots,NT-1$. For a chromosome Ch that represents a customer sequence, $G(Ch)$ is a directed graph, and its node set is $V(G)=\{g(i)\,|\,1\le i\le n\}$. Directed edge set is $E(G)$, if and only if it meets the constraints (16)-(17), $(i,j)\in E(G)$.

$$\sum_{m=i+1}^{j} UL_{g(m)} \le MC_k \tag{16}$$

$$\begin{aligned} &\forall m=\{i+1,\cdots,j\}: \\ &x_m^k = T_{0,g(i+1)}^k + \sum_{h=i+1}^{m} T_{g(h),g(h+1)}^k + \sum_{h=i+1}^{m-1} S_h \\ &U_{g(m)} \le x_m^k \le L_{g(m)} \end{aligned} \tag{17}$$

In a feasible route, the constraints of vehicular capacity and the time windows of the customers should be all fulfilled. In the following, $(i,j)\in E(G)$ is an edge and k is a vehicular type. Each arc (i,j) is a feasible distribution route if the vehicle left at the node 0 (depot) and continuously accesses the nodes $i+1,i+2,\cdots,j$ in turn. Total unloaded quantity (or served demand) is computed by $\sum_{m=i+1}^{j} UL_{g(m)}$; x_m^k is the time when the vehicle of the vehicular type k arrives at node m. The calculation approach of it is shown in Equation (17). The edge's weight $w_{i,j}$ defines the traveling costs of the vehicle of the vehicular type k that starts from the depot and accesses the nodes $i+1, i+2, \cdots, j$ in turn. If the Equations (16)-(17) are out of consideration, the weight matrix is given in Figure 1, where ∞ presents that the traveling cost is infinite. Other numerical sequence means the traveling costs of accessing the nodes in the sequence in turn.

$$Ch = \{g(1), g(2), g(3), \cdots, g(n)\} = \{2,1,6,5,4,3\}$$

$$\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{bmatrix} \infty & 0\text{-}2\text{-}0 & 0\text{-}2\text{-}1\text{-}0 & 0\text{-}2\text{-}1\text{-}6\text{-}0 & 0\text{-}2\text{-}1\text{-}6\text{-}5\text{-}0 & 0\text{-}2\text{-}1\text{-}6\text{-}5\text{-}4\text{-}0 & 0\text{-}2\text{-}1\text{-}6\text{-}5\text{-}4\text{-}3\text{-}0 \\ \infty & \infty & 0\text{-}1\text{-}0 & 0\text{-}1\text{-}6\text{-}0 & 0\text{-}1\text{-}6\text{-}5\text{-}0 & 0\text{-}1\text{-}6\text{-}5\text{-}4\text{-}0 & 0\text{-}1\text{-}6\text{-}5\text{-}4\text{-}3\text{-}0 \\ \infty & \infty & \infty & 0\text{-}6\text{-}0 & 0\text{-}6\text{-}5\text{-}0 & 0\text{-}6\text{-}5\text{-}4\text{-}0 & 0\text{-}6\text{-}5\text{-}4\text{-}3\text{-}0 \\ \infty & \infty & \infty & \infty & 0\text{-}5\text{-}0 & 0\text{-}5\text{-}4\text{-}0 & 0\text{-}5\text{-}4\text{-}3\text{-}0 \\ \infty & \infty & \infty & \infty & \infty & 0\text{-}4\text{-}0 & 0\text{-}4\text{-}3\text{-}0 \\ \infty & \infty & \infty & \infty & \infty & \infty & 0\text{-}3\text{-}0 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Figure 1 : Weight matrix

Because there may be more than one vehicular types that meet the above two constraints, the vehicular types with smaller capacity are considered with higher priority when they are chosen for routing. In other words, if a distribution task can be completed by a vehicle with smaller capacity, then other vehicles with larger capacity will not be considered. It can be assumed that the vehicles with smaller capacity has lower fixed costs and faster speed, and can save more space on the premise of satisfying customer demand. If larger vehicle is assigned, more space will be wasted. Obviously, the vehicles with lower costs will be chosen with higher priority. Therefore, the weight of arc is calculated by Equation (18). The vehicular type of the minimal cast is chosen by the equation: $k = \arg\min_k \{q \mid q \in TR, Eq(16\text{-}17)\}$.

$$\omega_{i,j} = D_{0,g(i+1)} + \sum_{h=i+1}^{j-1} D_{g(h),g(h+1)} + D_{g(j),0} \quad \textbf{(18)}$$

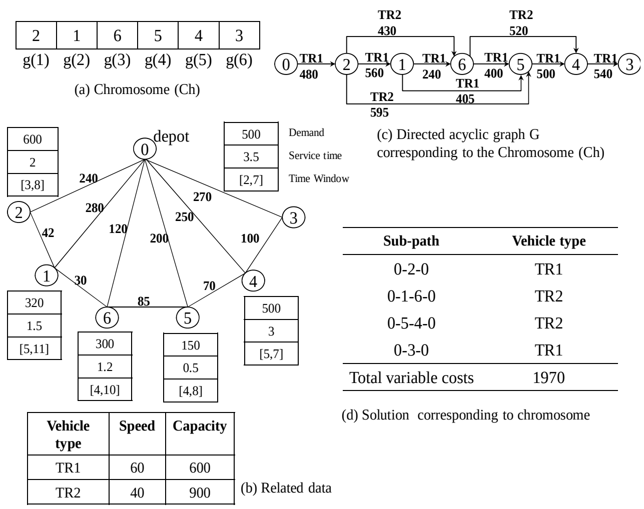
The shortest path in the graph $G(Ch)$ is an optimal partition of the customers, which is taken as the solution corresponding to the chromosome.

In the following, a sample is given. Given six customers and two vehicular types, the chromosome is denoted as $Ch = \{g(1), g(2), g(3), \cdots, g(n)\} = \{2,1,6,5,4,3\}$, as shown in Figure 2(a). Some input data are provided in Figure 2(b). The number on the arc (i, j) is the distance $D_{i,j}$; the number near the node i is the demand (UL_i), service time at the customer node (S_i) and the time window ($[L_i, U_i]$). According to the above information, a directed acyclic graph $G(Ch)$ is constructed as shown in Figure 2(c). For example, the arc $(g(1), g(3))=(2,6)$ presents the route of customer node sequence, $(0, g(2), g(3), 0) = (0,1,6,0)$. In the following, the vehicular types 1 and 2 are tested whether the capacity and time window are met. By computation, the vehicular type 1 does not fulfill the constraints. For vehicular type 2, the constraints are tested as follows.

1) $UL_1 + UL_6 = 320 + 300 \le 900$. Therefore, the capacity constraint is met.

2) $x_1^2 = 280/4 = 7 \in [5,11]$, and $x_6^2 = 280/40 + 1.5 + 30/40 = 9.25 \in [4,10]$. Therefore, the time window is satisfied.

Therefore, the arc $(g(1),g(3))=(2,6)\in G(Ch)$ is a feasible path for a vehicle. The weight of the arc is 280+30+120=430. For the direct graph $G(Ch)$, every path from the depot 0 to the final node $g(n)=3$ in the chromosome is a feasible solution that can meet the demands of the n customers. The decoding approach based on shortest path algorithm grantees that the solution with the lowest cost is obtained, and the solution is shown in Figure 2(d). Therefore, by this decoding approach, the vehicular types, the number of each vehicular type and the routes that consist of the nodes set and assessing sequence of customers are all determined.



Sub-path	Vehicle type
0-2-0	TR1
0-1-6-0	TR2
0-5-4-0	TR2
0-3-0	TR1
Total variable costs	1970

(d) Solution corresponding to chromosome

Vehicle type	Speed	Capacity
TR1	60	600
TR2	40	900

(b) Related data

Figure 2 : Sample of chromosome decoding

Comparing to the decoding process, the encoding process is relatively simple. Any permutation of the customer nodes can be a feasible chromosome.

Based on the above analysis, the decoding process is defined as Algorithm 2.

Algorithm 2 (Decoding algorithm)

Input: The chromosome $Ch=\{g(1),g(2),g(3),\cdots,g(n)\}$;

The distance between any two nodes, $D_{i,j}, i,j=\{0,1,\cdots,n\}$;

The customer demands UL_i, service time S_i, and time window $[L_i,U_i]$ for all $i\in\{1,\cdots,n\}$;

The capacity of vehicular type, MC_k for all $k\in TR=\{1\ ;\cdots,NT\}$.

Output: The solution after decoding the chromosome Ch: $PCh_l, TCh_l, l\in\{1,2,\cdots,NP\}$.

NP is the number of paths (also the number of the dispatched vehicles);

PCh_l is a sequence of customer nodes in a path;

$TCh_l\in TR$ is the vehicular type of the path.

Variables: $\Omega_{i,j},\omega_{i,j}, i,j=\{0,1,\cdots,n\}$: matrix of vehicle type and the matrix of traveling cost upon the directed graph $G(Ch)$ corresponding to the chromosome.

Vector $\alpha=(\alpha(1),\alpha(2),\cdots,\alpha(\mathrm{x}))$ that stores the result vectors of shortest path algorithm.

Process

Step 1. Initialization : Set $\Omega_{i,j}=0,\omega_{i,j}=M, i,j=\{0,1,\cdots,n\}$.

Step 2. For $\forall i=\{0,1,\cdots,n\}, j=\{i+1,\cdots,n\}$, $k=1, m=i+1$, Goto Step 3

Step 3. If $k>NT$, set $j=j+1$, Goto step 2;

Or else, if Equations (16)-(17) are satisfied, Goto Step 4;

Or else, set $k=k+1$ and Goto Step 3.

Step 4. Calculate

$\omega_{i,j} = D_{0,g(i+1)} + \sum_{h=i+1}^{j-1} D_{g(h),g(h+1)} + D_{g(j),0}$, $\Omega_{i,j}=k$.

For any $i=\{0,1,\cdots,n\}$ and $j=\{i+1,\cdots,n\}$ repeat Step 2 to Step 4, then Goto Step 5.

Step 5. According to the cost matrix of the directed graph $G(Ch)$, the shortest path algorithm Dijkstra's algorithm (see Xu et al. (2007)) is used to compute the shortest route between node 0 to node $g(n)$. The result vector is denoted as $\alpha=(\alpha(1),\alpha(2),\cdots,\alpha(\mathrm{x}))$, where x is the number of dispatched vehicles, $NP=x\text{-}1$. Goto Step 6.

Step 6. $\forall l=\{1,\cdots,NP=x\text{-}1\}$, calculate the following two outputs and return.

$TCh_l=\Omega_{l,l+1}$

$$PCh_l = \begin{pmatrix} 0, g(\alpha(l)), g(\alpha(l)+1),..., \\ g(\alpha(l+1)), 0 \end{pmatrix},.$$

(2) Population initialization

The population initialization is generated by random method in this paper. Each chromosome is a permutation of the integers from 1 to n. Therefore, the initial population consist of *popsize* permutations. The proper setting of *popsize* affects the computational performance. The larger the population size, the longer the searching process for optimal solution. However, small size population may trap into local optimum.

(3) Genetic operators

Tournament selection operator, Partially Matched Exchange (PMX) operator and bit mutation operator, and rank-based based on selection scheme are used in Algorithm 1.

(4) Fitness function

The optimization objective of FSMVRPQD is the minimization of costs. Using Equation (20), the objective function is changed into fitness function, when z_i is the objective value corresponding to the chromosome i in the population, namely the total outsourcing costs with quantity discounts. The fitness value corresponding to the chromosome i determines the probability of the chromosome to produce offspring.

$$f_i = \frac{1}{z_i}, i=\{1,\cdots,popsize\} \quad \textbf{(20)}$$

(5) Terminal condition

The number of generations of evolution is used as termination rules. When the algorithm has been iterated for the time of the generation number, the algorithm stops and returns the elite as the final solution. The path sets that are decoded from the elite chromosomes are obtained as the optimal solution of the FSMVRPQD.

DEMONSTRATION

In Figure 3, one hundred of demand points and their distribution are shown. The point (numbered as 1) is chosen as the depot. The demand and time window of each point are shown in TABLE 1. The speed, capacity, cost and outsourcing discounts of vehicles of three types (classified by the capacity, small, medium and large) are shown in TABLE 2.

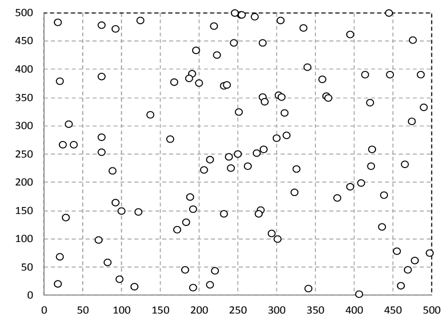


Figure 3 : Distribution of demand points

TABLE 1 : The demand and time window of customer points

Point	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Demand	4	5	10	9	9	5	2	7	10	1	2	6	10	4	1	8	5	4	8	7
L	0	7	12	3	3	0	12	20	2	14	15	0	10	17	16	3	8	8	6	15
U	M	29	47	49	32	M	46	68	18	20	51	M	43	59	28	40	44	54	38	51
Point	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Demand	6	5	2	10	7	1	6	2	7	3	8	10	6	5	5	5	6	8	1	4
L	7	9	18	18	0	16	5	12	14	5	18	5	17	13	4	14	5	10	1	15
U	52	55	62	63	M9	61	36	61	44	14	38	35	59	54	33	36	27	21	44	46
Point	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Demand	2	10	2	4	3	1	8	10	2	6	6	7	3	10	1	1	1	8	2	5
L	18	13	1	0	0	5	5	18	15	10	16	15	0	19	8	3	14	9	10	7
U	54	40	41	M	M	10	44	68	22	46	38	64	M	24	13	25	52	34	13	47
Point	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Demand	1	8	10	3	6	9	9	4	2	4	4	5	1	6	5	6	3	6	1	6
L	11	20	10	0	0	8	4	8	0	4	8	1	18	0	12	0	0	0	6	7
U	21	68	53	M	M	56	29	39	M	40	50	14	47	M	48	M	M	M	22	26
Point	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
Demand	9	9	6	10	7	6	8	3	6	10	7	5	4	6	8	5	6	4	7	3
L	13	20	12	13	0	0	14	20	20	15	10	11	11	18	19	12	20	11	0	0
U	63	29	61	24	M	M	27	45	27	42	47	35	58	60	58	34	44	17	M	M

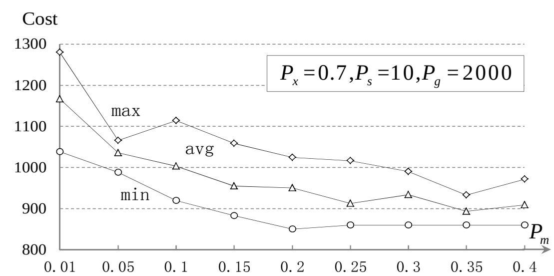
Note: M is infinite number

TABLE 2 : Vehicular type and discount information

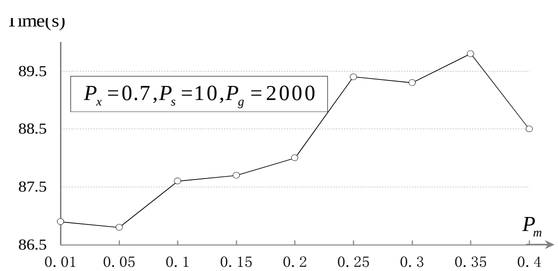
No.	**Type**	**Speed**	**Capacity**	**Cost**	**Quantity interval of outsource at discounts**		
					10	**9**	**8**
1	small	80	20	20	[1,8]	[9,15]	15
2	medium	60	40	50	[1,6]	[7,12]	12
3	large	40	60	80	[1,4]	[5,9]	9

The Algorithm 1 and Algorithm 2 are used to solve the case. The algorithms are implemented with C# and ran on the computer with CPU of Intel core i3-380 M and 2GB RAM. In order to study the influences of each parameter on the experiment results and the run time, different combinations of parameters are set, and ten times are run for each parameter settings. The comparison analysis of experimental results and the run time are shown in Figure 4.

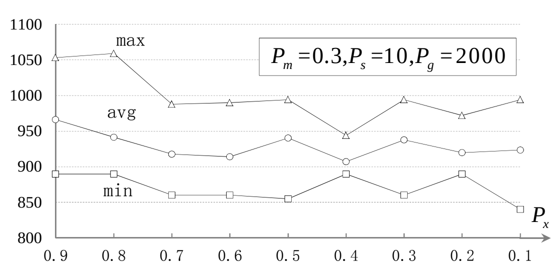
Figures 4(a1), 4(b1) and 4(c1) are the results under different mutation probability P_m, crossover probability P_x and population size P_s. Three curves respectively represent the maximum, average value, minimum of the ten times of experiments with different parameter settings. Figure 4(a2), 4(b2) and (c2) are average run time corresponding to the experiments of Figures 4(a1), 4(b1) and 4(c1). It is shown in Figure 4 that on the premise of other parameters are certain, the optimal costs decrease generally with the increase of P_m. When P_m is 0.35, the ten experiments get the lowest maximum, average value, minimum value. P_x has no clear influence to the experiments, and when it increase, the costs shows a decrease tendency. Furthermore, as shown in Figure 4(a2) and 4(b2), the curves show no obvious tendency and laws, and the maximal and minimal average run time for the experiments are only a difference of 3 second and 2.2 seconds, which can be ignored relative to the total average run time of 88.22 and 88.34. Therefore, it is concluded that P_x and P_m have no significant influence on the run time. It can be directly seen from Figure 4(c3) that the curve is almost a straight line rising perpendicularly. When P_s increases by 10, the average operation time almost increases by 100s. Therefore, it can be concluded that the running time relates to the P_s positively.



(a1) Optimal cost under mutation probability



(a2) Run time under mutation probability



(b1) Optimal cost under crossover probability

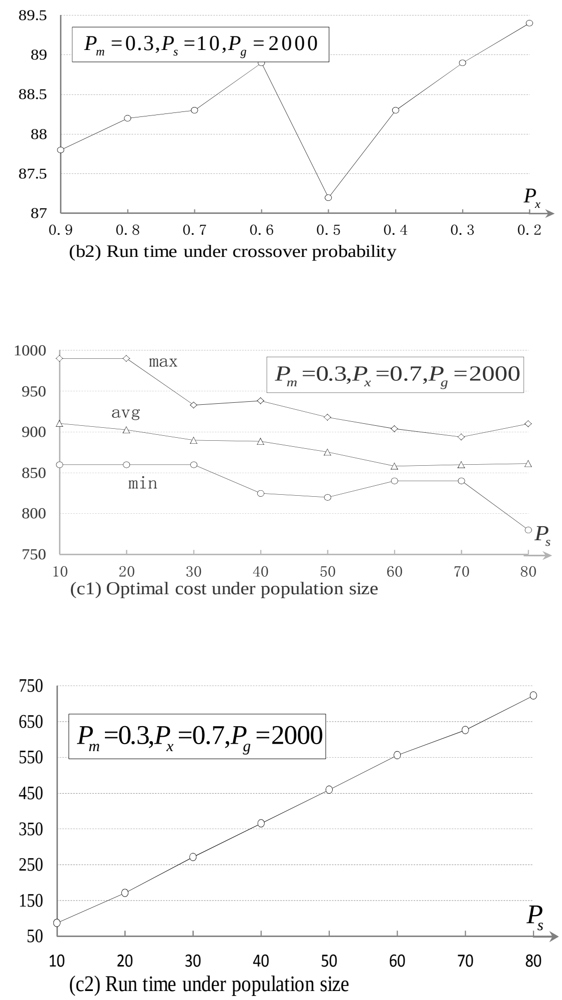


Figure 4 : Experiment results and run time under different parameter settings

According to the above analysis and take the experiment result and the run time into account together, the parameters are chosen to set as $P_s = 60$, $P_x = 0.7$, $P_m = 0.35$ and $P_g = 10000$. The algorithm has been run 5 times with an average run time of 46 minutes and 50 seconds, 4 times of which are converge to the optimal cost 780 after 10000 generations of evolution.

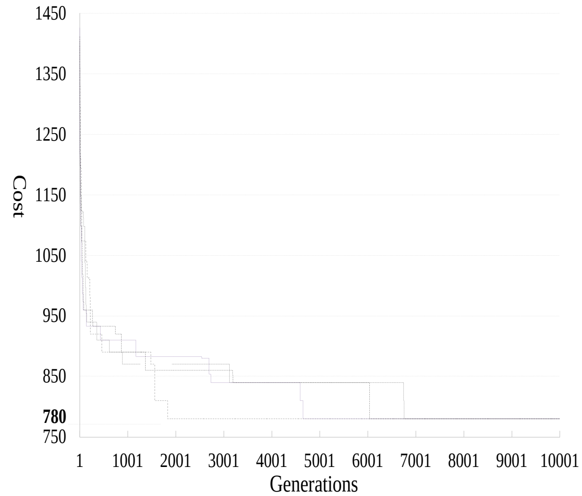


Figure 5 : Evolution trend of solution with generations

As shown in Figure 5, at the beginning, the curve decreases rapidly. With the process of evolution, it tends to be gentle, but the convergence trend still can be seen obviously from the evolution process.

In particular, though the optimal values of four times of experiments are the same, the distribution solution of routes and vehicles may be different. Because the FSMVRPQD uses outsourcing cost to measure the fitness of GA, which only determined by the amount and type of the outsourcing vehicles and has nothing to do with the distribution paths. According to these characteristics, when objective values of the multiple solutions are the same, solutions with minimal variable cost (measured with traveling distance) can be chosen. Therefore, comparing to the total traveling distances of the four times of experiments, the best experiment reaches the minimal traveling distance of 24177 after 4653 generations. The final optimal distribution solution is shown in TABLE 3, where the number of vehicles is 14 and the total cost of outsource is 780.

TABLE 3 : Final distribution solution

No.	D	VT	Path (customer node sequence)
1	50	3	0-29-8-13-5-2-20-36-0
2	44	3	0-90-17-22-50-70-96-66-0
3	50	3	0-85-63-53-88-97-28-51-18-10-34-45-0
4	23	2	0-78-54-91-0
5	47	3	0-71-52-81-65-80-74-82-0
6	59	3	0-41-16-75-86-62-21-95-9-94-0
7	51	3	0-99-26-38-76-24-31-14-15-33-0
8	33	3	0-73-6-4-47-42-0
9	51	3	0-93-87-68-1-60-19-58-48-0
10	2	1	0-69-0
11	46	3	0-11-32-64-83-84-46-98-61-67-0
12	20	1	0-59-92-77-57-100-40-23-0
13	53	3	0-56-39-3-49-30-55-79-44-72-89-12-

			37-35-7-0
14	13	1	0-27-43-25-0

Note : D=Demand, VT=Vehicle type

CONCLUSION

Comparing to the research results on typical mixed vehicle routing problems, it is a strategy to reduce outsourcing cost, improve the fleet profits and reduce the empty loading rate by considering the time window constraint and quantity discounts, and incorporating the financial concepts to vehicle routing optimization problem. Because the mixed vehicle routing problem with time window constraint and quantity discounts is a nonlinear programming model, an evolutionary algorithm is designed in this paper based on the decoding approach that uses the shortest path algorithm to generate routes from chromosome. During the decoding process and in the fitness calculation method, an overall consideration of the benefits of the fleet and the customers is taken into account by considering the fixed costs and variable costs of the fleet and the customers together. In algorithmic experiments, through parameter tests it is found that the total cost appears to be decline in general with the increase of mutation probability and population size, while the crossover probability has no significant effect to the search of the optimums. The population size influences the run time. When the population size is increased by 10, the average computational time will be increased by about 100s. Moreover, mutation probability and crossover probability have no significant effect on the run time. Through setting the optimal parameters of the algorithm, the optimal distribution schedule is obtained by comparison analysis of the results. Several points are to be further studied. Local search methods can be incorporated to improve the algorithmic performance; the equilibrium relationships among different costs can be analyzed by considering multi-objective optimization; in addition, the effects of vehicular type to transport costs and the influence of discount are all problems worth of study and discuss in the future.

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