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THERMAL EFFECTS IN NON-NEWTONIAN LUBRICATION OF ASYMMETRIC ROLLERS UNDER ADIABATIC AND ISOTHERMAL BOUNDARIES

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ABSTRACT

A theoretical aspect of thermal effects in hydrodynamic lubrication of roller bearings is analyzed under adiabatic and isothermal boundaries for heavily loaded rigid system. Basically, it describes a qualitative analysis of system with non-Newtonian incompressible power law fluids, in which the consistency of the lubricant is assumed to vary with the pressure and the mean film temperature. The fluid flow governing equations such as equation of motion with continuity and momentum energy equations are solved analytically first and then finally numerically by Runge-Kutta Fehlberg method. The various important characteristics of bearings are discussed and elaborated through graphs. As a nut cell, a significant change in pressure, temperature, load and traction with Newtonian and non-Newtonian fluids are characterized.

Key words: Hydrodynamic lubrication, Thermal effects, Roller bearings, Non-newtonian, Power law, Incompressible.

INTRODUCTION

In the recent years, research with temperature effect in lubrication has received much attention. In fact, it is the pressure and the temperature, which play vital role in the failure of heavily loaded contacts. Hence, the effect of heat generated due to the shearing of the high-pressure lubricant, is no longer negligible under the rolling and sliding conditions as the heat changes the characteristics of the oil flow[1]. The role of the pressure and the temperature becomes even more severe, when the bearing is running at heavy loads and high speed, and finally causing some significant change in the lubricant material properties[2,3].

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Anandan et al.[4] presented some empirical relations for the prediction of thermal minimum film thickness, maximum mid-film temperature rise, and rolling/sliding traction coefficient for lubricated line contact problem, applicable for fully flooded and starved conditions. Recently, Kumar et al.[5] analyzed a similar problem, where the changes in viscosity and density of the lubricant with pressure and temperature both were taken into account. Later, Sojoudi & Khonsari[6] proposed a dynamic friction model for predicting the friction coefficient under both quasi steady and unsteady operating conditions. Recently, Vengudusamy et al.[7] investigated the temperature influence on the friction performance of five different gear oils in rolling-sliding and pure sliding contacts, and found that increasing temperature decreases boundary friction with gear oils that contain friction modifiers while not for other gear oils. Also the effect of slide-roll ratio on friction is significant in boundary lubrication region especially at high temperature while less significant in mixed lubrication region at both low and high temperatures.

In most of the above problems, lubricant was assumed to be Newtonian. The non-Newtonian characteristics have also been invariably served in various lubrication problems. The use of non-Newtonian fluid models in describing the squeezing flow in hydrodynamic lubrication containing solid particle additives has received a great attention[8]. In fact, the additives present in lubricant improve and strengthen its life, in turn, the lubricant protects and extends the life of equipment. Additives can improve the physical properties of the lubricant while increasing its performance[9].

The non-Newtonian power law model has got attentions in the recent years[10]. Temperature rise in lubricant film occurs due to rapid shear of lubricant layer. The hydrodynamic action is affected due to thermal effect. Such temperature rise for a non-Newtonian lubricant can be better estimated using power law model[11]. Prasad et al.[12] examined the problem of pure rolling of cylinders considering the power law lubricant with thermal and compressibility effect under adiabatic condition where heat of convection was assumed to be dominant to that of conduction. Further, Prasad et al.[13] studied thermal lubrication of rolling/sliding contacts by an incompressible power law fluid and they concluded that there is a significant change in pressure & the mean temperature with respect to power law index and roll ratio. Later, Prasad et al.[14] presented a theoretical analysis of symmetrical cylindrical rollers lubricated with power law fluids. Thermal and inertia effects have also been incorporated together with squeezing motion and film cavitations. Recently, Prasad and Subrahmanyam[15] extended some study of hydrodynamic lubrication of asymmetric rollers assuming the consistency of the non-Newtonian incompressible power law lubricants to vary with pressure and the mean film temperature under isothermal boundaries. Kumar et al.[5] investigated the effect of polymeric fluid additives on EHL behavior of rolling/sliding line contacts at low as well as high speeds. The results show a significant variation in maximum fluid pressure and minimum fluid film thickness with the volume fraction, referencing viscosity ratio and power law index of the polymeric fluid additive. Almqvist et al.[16] proposed a theory based on clear physical arguments related to conservation of mass flow and considers both incompressible and compressible fluids. The result of the mathematical modeling makes a system of equations with two unknowns, which are related to the hydrodynamic pressure and the degree of saturation of the fluid. The model and the associated numerical solution method have significant advantages over today's most frequently used cavitation algorithms, which are based on Elrod–Adams pioneering work.

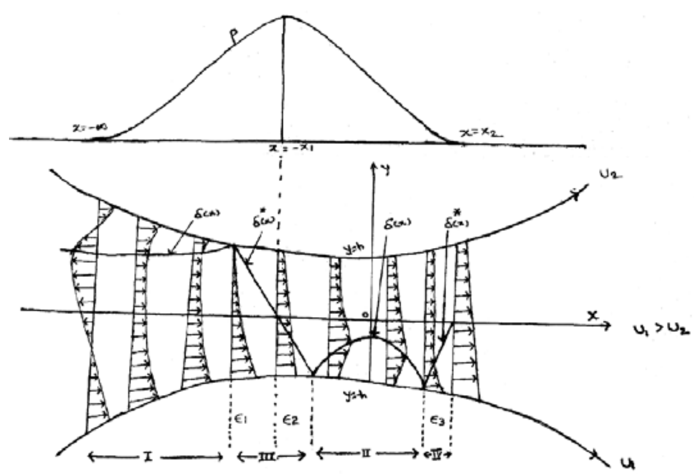

Fig. 1: Lubrication of asymmetric rollers

In the present analysis, an investigation is fully concentrated to study the qualitative behavior of thermal effects on non-Newtonian power law lubrication of two heavily loaded rigid asymmetric rollers under adiabatic and isothermal boundaries. The consistency variation of the lubricant is assumed to vary with the pressure and the mean film temperature; however, the effects of compressibility and surface roughness are neglected.

Mathematical formulations

Fluid flow governing equations

The fluid flow equations of the hydrodynamic lubrication with some usual assumptions are[13].

$$\frac{dp}{dx} = \frac{\partial}{\partial y}\left[m \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right] \quad \ldots(1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \ldots(2)$$

where
$$m = m_0 e^{\alpha p-\beta(T_m-T_0)} \qquad \ldots(3)$$

with
$$T_m = \frac{1}{2h}\int_{-h}^{h} T\, dy \qquad \ldots(4)$$

and
$$h = h_0 + \frac{x^2}{2R} \qquad \ldots(5)$$

R is being the radius of the equivalent cylinder.

Boundary conditions

$$u = U_1 \text{ at } y=-h;\text{ and } u=U_2 \text{ at } y=h \qquad \ldots(6)$$

$$p=0 \text{ at } x=-\infty \qquad \ldots(7)$$

$$p=0 \text{ and } \frac{dp}{dx}=0 \text{ at } x=x_2 \qquad \ldots(8)$$

Using the sign of the pressure gradients (as given in Fig. 1), one may obtain respectively.

$$\left.\begin{aligned} u_1 &= U_2+\left(\frac{n}{n+1}\right)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n}\left[(y-\delta)^{\frac{n+1}{n}}-(h-\delta)^{\frac{n+1}{n}}\right], \delta\le y\le h\\ u_2 &= U_1+\left(\frac{n}{n+1}\right)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n}\left[(\delta-y)^{\frac{n+1}{n}}-(\delta+h)^{\frac{n+1}{n}}\right], -h\le y\le \delta \end{aligned}\right\} \qquad \ldots(9)$$

$$\left.\begin{aligned} u_3 &= U_1+\left(\frac{n}{n+1}\right)\left(-\frac{1}{m_2}\frac{dp_2}{dx}\right)^{1/n}\left[(\delta+h)^{\frac{n+1}{n}}-(\delta-y)^{\frac{n+1}{n}}\right], -h\le y\le \delta\\ u_4 &= U_2+\left(\frac{n}{n+1}\right)\left(-\frac{1}{m_2}\frac{dp_2}{dx}\right)^{1/n}\left[(h-\delta)^{\frac{n+1}{n}}-(y-\delta)^{\frac{n+1}{n}}\right], \quad \delta\le y\le h \end{aligned}\right\} \qquad \ldots(10)$$

Now the volume flux Q for the region: $-\infty < x < -x_1$, is

$$Q = \int_{-h}^{h} u\, dy$$

$$= (U_1 + U_2)\, h + (U_1 - U_2)\, \delta - \left(\frac{n}{2n+1}\right)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n}\left[(h-\delta)^{\frac{2n+1}{n}} + (h+\delta)^{\frac{2n+1}{n}}\right] \quad \ldots(11)$$

Similarly $$Q(-x_1) = (U_1 + U_2)\, H \quad \ldots(12)$$

where film thickness H at $x = -x_1$ is given by –

$$H = 1 + x_1^{\,2} \quad \ldots(13)$$

Reynolds equation

Then, equating the flux (11) and (12), and simplifying them, it can be written as –

$$\frac{dp_1}{dx} = m_1\left(\frac{2n+1}{n}\right)^n\left[\frac{(U_1+U_2)(h-H) + (U_1-U_2)\delta}{(h+\delta)^{\frac{2n+1}{n}} + (h-\delta)^{\frac{2n+1}{n}}}\right]^n, \; -\infty < x \le -x_1 \quad \ldots(14)$$

$$\frac{dp_2}{dx} = -m_2\left(\frac{2n+1}{n}\right)^n\left[\frac{(U_1+U_2)(H-h) - (U_1-U_2)\,\delta}{(h+\delta)^{\frac{2n+1}{n}} + (h-\delta)^{\frac{2n+1}{n}}}\right]^n, \; -x_1 \le x \le x_2 \quad \ldots(15)$$

Using the velocity matching conditions at y = δ; one can get –

$$(U_1 - U_2) + E\left[\frac{(U_1+U_2)(h-H) + (U_1-U_2)\delta}{(h+\delta)^{\frac{2n+1}{n}} + (h-\delta)^{\frac{2n+1}{n}}}\right]\left[(h-\delta)^{\frac{n+1}{n}} - (h+\delta)^{\frac{n+1}{n}}\right] = 0 \quad \ldots(16)$$

Heat energy equation

The heat energy equation with usual assumptions is considered to be[17-19]:

$$\frac{\rho\, c}{k}\left(u_m \frac{dT_m}{dx}\right) = \frac{\partial^2 T}{\partial y^2} + \left(\frac{m}{k}\right)\left|\frac{\partial u}{\partial y}\right|^{n-1}\left(\frac{\partial u}{\partial y}\right)^2 \quad \ldots(17)$$

With $u\dfrac{\partial T}{\partial x} \approx u_m \dfrac{dT_m}{dx}$ [18]

$$\frac{\partial T}{\partial y} = 0 \text{ at } y = h \text{ and } T = T_{-h} \text{ at } y = -h \qquad \ldots(18)$$

Now, solving (17) using (18) in conjunction with (4), the mean temperature T_{m1}, and T_{m2} can be calculated as:

$$T_{m1} = \frac{\varphi_1(x)h^2}{6} + \frac{nS_1}{2h(4n+1)}\left[(h-\delta)^{\frac{4n+1}{n}} + (h+\delta)^{\frac{4n+1}{n}}\right] - a\,, \quad -\infty < x \le -x_1 \qquad \ldots(19)$$

$$T_{m2} = \frac{\varphi_2(x)h^2}{6} + \frac{nS_2}{2h(4n+1)}\left[(h-\delta)^{\frac{4n+1}{n}} + (h+\delta)^{\frac{4n+1}{n}}\right] - b\,, \quad -x_1 \le x \le x_2 \qquad \ldots(20)$$

Where $a = \frac{3h^2}{2}\varphi_1(x) + S_1\left[\left(\frac{2}{E_5}\right)h(h-\delta)^{\frac{2n+1}{n}} + (h+\delta)^{\frac{3n+1}{n}}\right] - T_{-h}$;

$$b = \frac{3h^2}{2}\varphi_2(x) + S_2\left[\left(\frac{2}{E_5}\right)h(h-\delta)^{\frac{2n+1}{n}} + (h+\delta)^{\frac{3n+1}{n}}\right] - T_{-h}$$

$$S_1 = \left(\frac{m_1}{k}\right)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{\frac{n+1}{n}}\left(\frac{-n^2}{(2n+1)(3n+1)}\right); \text{ and}$$

$$S_2 = \left(\frac{m_2}{k}\right)\left(-\frac{1}{m_2}\frac{dp_2}{dx}\right)^{\frac{n+1}{n}}\left(\frac{-n^2}{(2n+1)(3n+1)}\right)$$

Now, using the dimensionless scheme[15], the equations (14), (15), (16), (19), and (20) can be rewritten as –

$$\frac{d\bar{p}_1}{d\bar{x}} = \bar{m}_1(\bar{f}_x)^n \qquad \ldots(21)$$

$$\frac{d\bar{p}_2}{d\bar{x}} = -\bar{m}_2(-\bar{f}_x)^n \qquad \ldots(22)$$

$$(\bar{U} - 1) + E\,\bar{J}_{1x}\,\bar{f}_x = 0 \qquad \ldots(23)$$

$$\frac{4}{3}\overline{p_e}\,\bar{h}^2\bar{u}_m\frac{d\bar{T}_{m1}}{d\bar{x}} + \bar{T}_{m1} - \bar{T}_{-h} - E_5\,\bar{\gamma}\,\bar{m}_1(\bar{f}_x)^{n+1}\overline{g_x} = 0 \qquad \ldots(24)$$

$$\frac{4}{3}\overline{p}_e\overline{h}^2\overline{u}_m\frac{d\overline{T}_{m2}}{d\overline{x}}+\overline{T}_{m2}-\overline{T}_{-h}-E_5\overline{\gamma}\,\overline{m}_2(-\overline{f}_x)^{n+1}\overline{g}_x=0 \quad \ldots(25)$$

Where $\overline{f}_x=\dfrac{(\overline{U}+1)(\overline{h}-\overline{H})+(\overline{U}-1)\overline{\delta}}{(\overline{h}+\overline{\delta})^{\frac{2n+1}{n}}+(\overline{h}-\overline{\delta})^{\frac{2n+1}{n}}}$ and

$$\overline{g}_x=\left(\frac{2}{E_5}\right)\overline{h}(\overline{h}-\overline{\delta})^{\frac{2n+1}{n}}+(\overline{h}+\overline{\delta})^{\frac{3n+1}{n}}-\frac{n\overline{J}_{2x}}{2\overline{h}(4n+1)}$$

Load and traction

The load and the Tractions in dimensionless form are given by –

$$\overline{W}=\int_{-\infty}^{\overline{x}_2}\overline{p}\,d\overline{x}=-\int_{-\infty}^{\overline{x}_2}\overline{x}\frac{d\overline{p}}{d\overline{x}}d\overline{x} \quad \ldots(26)$$

$$\overline{T}_{Fh-}\left(=\alpha\frac{T_{F0}}{h_0}\right)=-\int_{-\infty}^{\overline{x}_2}\overline{\tau}_{\overline{y}=-\overline{h}}\,d\overline{x} \quad \ldots(27)$$

Similarly,

$$\overline{T}_{Fh+}=-\int_{-\infty}^{\overline{x}_2}\overline{\tau}_{\overline{y}=\overline{h}}\,d\overline{x} \quad \ldots(28)$$

RESULTS AND DISCUSSION

A semi analytical solution[15] of the Reynolds equations (21, 22) and the energy equations (24, 25) are obtained for the following representative values: U_2 = 400 cm/s, $h_0 = 4 \times 10^{-4}$ cm, $\alpha = 1.6 \times 10^{-9}$ dyne^{-1} cm^2, R = 3 cm, $\overline{\gamma}$ = 1000.5, $\overline{T}_{-h}$ =1.4; the sliding parameter $\overline{U}$ is chosen between 1.0 and 2.0.

Pressure profile

The pressure distribution $\overline{p}$ is a function of $\overline{U}$ and 'n', presented in Figs. 2 and 3. Fig. 2 shows the pressure profile $\overline{p}$ increases almost everywhere with rolling ratio $\overline{U}$ for fixed n. It can be seen from Fig. 3 that $\overline{p}$ increases with n for fixed $\overline{U}$. This kind of the behavior was observed by Wang et al.[20], Jang & Khonsari[21], Prasad et al.[22], Rong-Tsong & Hamrock[23], and Sajja and Prasad[24].

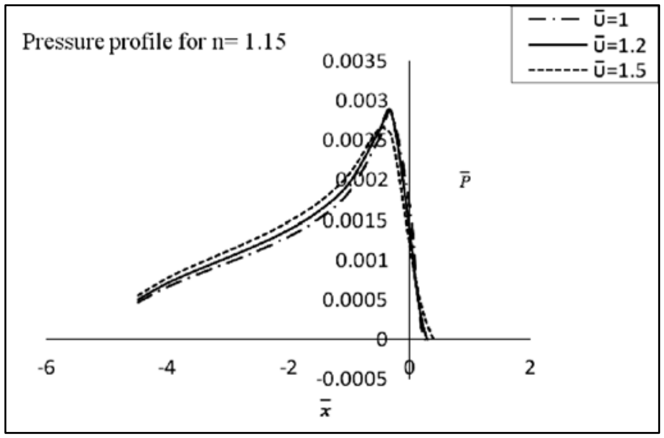

Fig. 2: Pressure $\overline{p}$ versus $\overline{x}$

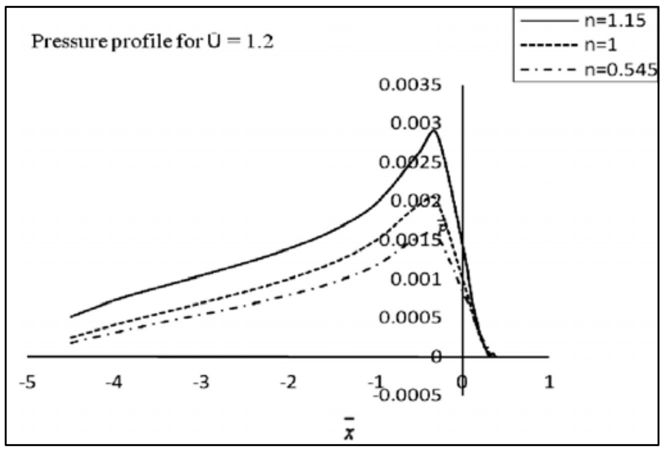

Fig. 3: Pressure $\overline{p}$ versus $\overline{x}$

Mean temperature profile

The mean temperature $\overline{T}_m$ are presented in Figs. 4, 5 and 6 for various values of n, $\overline{U}$ and $\overline{P}_e$. The mean temperature $\overline{T}_m$ increases with n showing that the temperature for dilatants fluid is higher than that of Newtonian and pseudo plastic fluids both[12]. The qualitative behaviors of $\overline{T}_m$ versus $\bar{x}$ is very similar to the temperature profile obtained by Prasad et al.[13], Sajja and Prasad[24] and Saini et al.[25] For fixed n and $\overline{P}_e$, $\overline{T}_m$ increase with $\overline{U}$, see Fig. 6. This indicates that the sliding temperature is higher than that of pure rolling (Ghosh and Hamrock[26], Sadeghi and Dow[27]). It may be here noted that the temperature $\overline{T}_m$ when $\overline{P}_e \to 0$ refer to the case without convection[28].

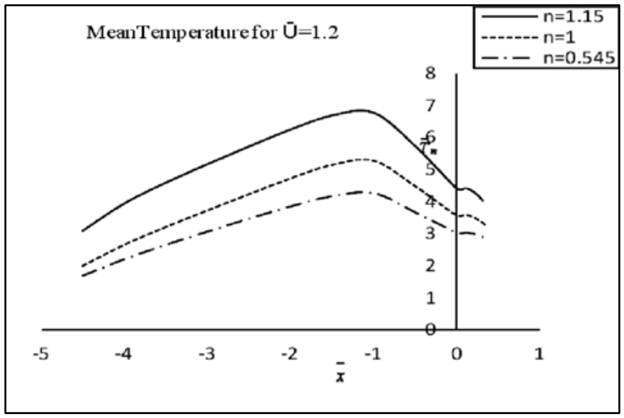

Fig. 4: Mean temperature $\overline{T}_m$ versus $\bar{x}$

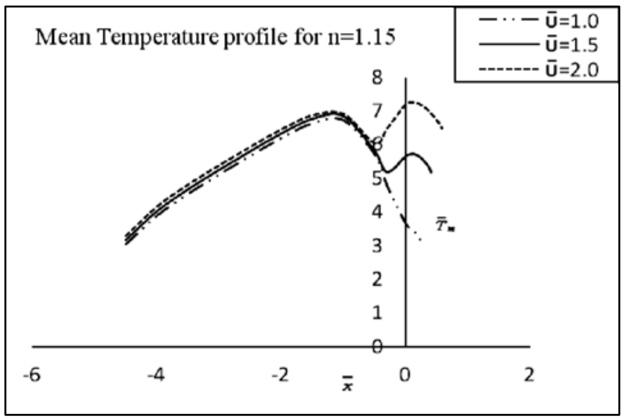

Fig. 5: Mean temperature $\overline{T}_m$ versus $\bar{x}$

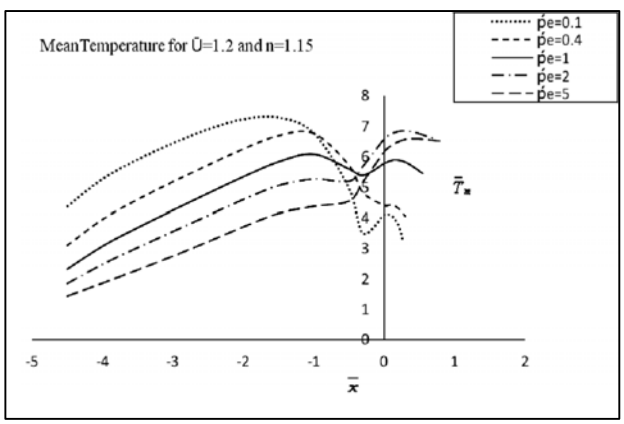

Fig. 6: Mean temperature $\overline{T}_m$ versus $\bar{x}$

Consistency profile

The consistency profile $\overline{m}$ is shown in Fig. 7 for different values of n keeping $\overline{U}$ fixed. One can observe that the consistency increases with n for a fixed value of $\overline{U} = 1.2$. The same trend is obtained by Prasad et al.[14] Qualitatively, $\overline{m}$ decreases in the pressure peak region then it increases up to $\overline{x} = 0$ and later decreases a little, and again increases up to the cavitation point. A similar trend was reported by Bruyere et al.[29].

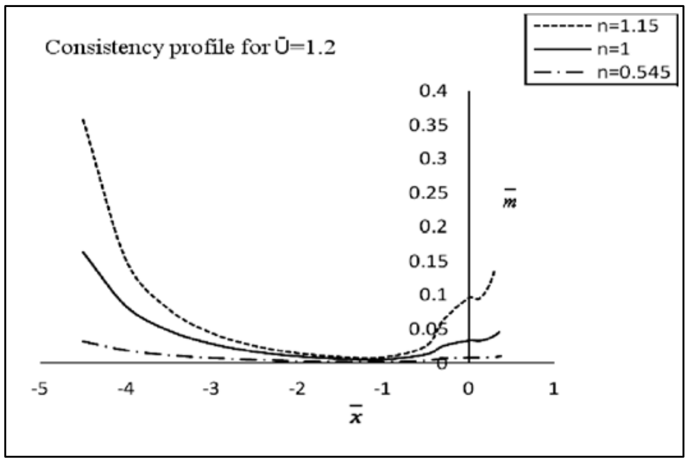

Fig. 7: Consistency $\overline{m}$ versus $\overline{x}$

Load and traction

The calculated values of the normal load carrying capacity $\overline{W}$, the traction force $\overline{T}_{Fh+}$ at $\overline{y} = \overline{h}$ and $\overline{T}_{Fh-}$ at $\overline{y} = -\overline{h}$ are presented respectively in Figs. 8, 9 and 10. It can be seen from Fig. 8 that $\overline{W}$ increases with n and $\overline{U}$ both, this is in consistent with the previous findings mentioned in[12,13].

The traction forces $\overline{T}_{Fh+}$ and $\overline{T}_{Fh-}$ have been evaluated for both the surfaces for various values of n & $\overline{U}$. These two also have the same characteristics as that of $\overline{W}$, that is, both increase with n and $\overline{U}$. The trends of the traction force with n and $\overline{U}$ are quite similar to that of Kumar and Khonsari[30]; and also similar to the prediction made by Durak et al.[9] for the journal bearing. Further, it is observed that as $\overline{U} \rightarrow 1$, both the traction forces approach towards the same value; this is on the line of our expectation also, since once $U_1 \sim U_2$, both the surfaces will experience the same drag force.

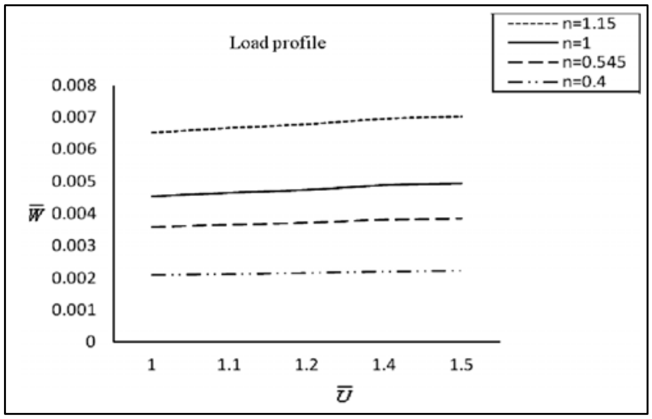

Fig. 8: $\overline{\mathbf{W}}$ **versus** $\overline{\mathbf{U}}$

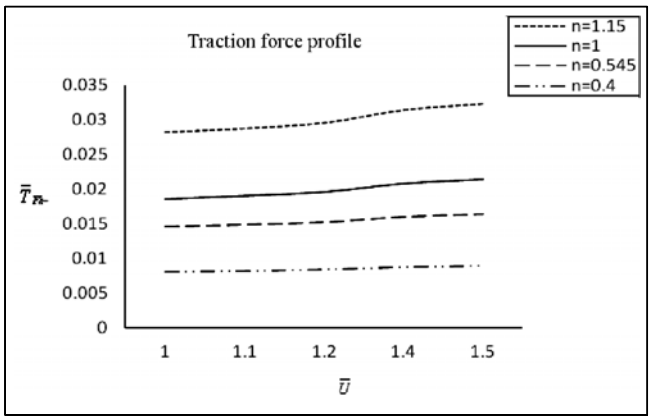

Fig. 9: $\overline{\mathbf{T}}_{\mathbf{Fh-}}$ **versus** $\overline{\mathbf{U}}$

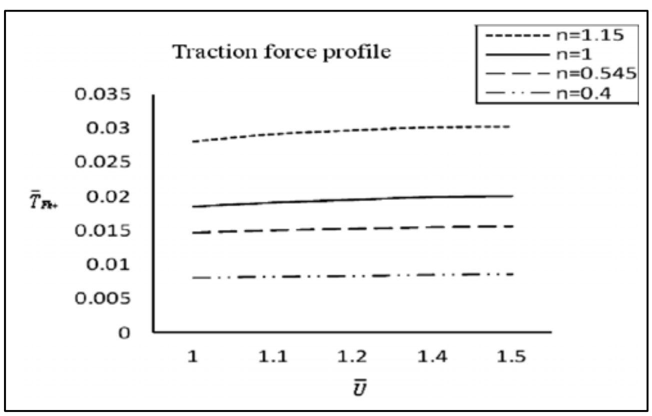

Fig. 10: $\overline{\mathbf{T}}_{\mathbf{Fh+}}$ **versus** $\overline{\mathbf{U}}$

$\bar{\delta}$ - Profile

The rough sketch of δ-profile is presented in Fig. 1 and numerically computed δ-profile is presented in Fig. 11. A similar profile has been given by Prasad et al.[13] This profile also looks like pressure profile presented by Morales-Espejel and Wemekamp[31].

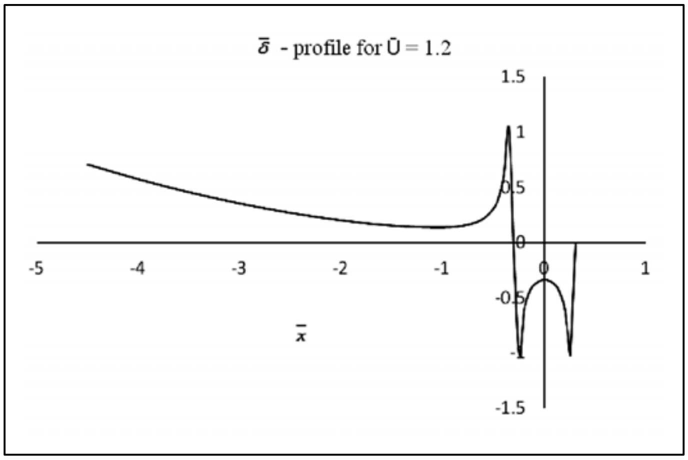

Fig. 11: Delta profile versus $\bar{x}$

Nomenclature

E : $\left(\frac{2n+1}{n+1}\right)$

E_5 : $\frac{2n}{3n+1}$

h : Lubricant film thickness

h_1 : Film thickness at x= - x_1

h_o : Minimum film thickness

$\bar{h}$: h / h_o etc.

$\bar{J}_{1x}$: $(\bar{h}-\bar{\delta})^{\frac{n+1}{n}} - (\bar{h}+\bar{\delta})^{\frac{n+1}{n}}$

$\bar{J}_{2x}$: $(\bar{h}-\bar{\delta})^{\frac{4n+1}{n}} + (\bar{h}+\bar{\delta})^{\frac{4n+1}{n}}$

m : Lubricant consistency etc.

m_o : Consistency at ambient pressure and temperature

$\bar{m}$: 2 m c_n α etc.

$\overline{m}_1$	: $\overline{m}_0\ e^{(\overline{p}_1 - \overline{T}_{m1} + \overline{T}_0)}$
n	: Consistency index of the power law lubricant
p	: Hydrodynamic pressure
$\overline{p}$	: αp etc.
Q	: Volume flux of the fluid
R	: Radius of the equivalent cylinder
T	: Lubricant film temperature
T_m	: Mean film temperature
T_0	: Ambient temperature
$\overline{T}$	: β T etc.
T_{Fh}	: Traction force etc.
$\overline{T}_{Fh}$	: Dimensionless traction force (= - (2α T_{Fh} / h_o)) etc.
U_1 , U_2	: Velocities of the cylinders at y = -h and y = h
u	: Velocity of the lubricant in x-direction
$\overline{u}$	: $\frac{u}{U_1}$ etc.
u_m	: The mean velocity of the lubricant $\left(= \frac{U_1 + U_2}{2}\right)$
v	: Velocity of the lubricant in y-direction
W	: Load in y-direction
$\overline{W}$	: Dimensionless load (= α W/$(Rh_o)^{½}$)
x,y	: Co-ordinate axes
$\overline{x}$	: Dimensionless distance in x-direction (= x/ $(2Rh_o)^{½}$)
x_1	: Point of maximum pressure
x_2	: Cavitation point
α	: Pressure coefficient
β	: Temperature coefficient
δ	: Location of points where velocity gradient $\frac{\partial u}{\partial y} = 0$
$\overline{\delta}$	: δ/ h_o

$\overline{\gamma}$: $\left(\frac{U_2 h_0 \beta}{2k\alpha}\right)\sqrt{\frac{h_0}{2R}}$

$\overline{p_e}$: $\frac{\rho c U_2 h_0}{k}\sqrt{\frac{h_0}{2R}}$

ϕ : $\frac{\rho c u_m}{k}\left(\frac{dT_m}{dx}\right)$ etc.

CONCLUSION

The problem consists of thermal effects in hydrodynamic lubrication of roller bearings by an incompressible power law fluid under adiabatic and isothermal boundaries. The Reynolds and the thermal energy equations, which are functions of consistency $\overline{m}$, sliding parameter $\overline{U}\,(= U_1/U_2)$ and the consistency index n are derived and solved semi-analytically for the pressure $\overline{p}$ and the mean temperature $\overline{T}_m$. The following inferences may be drawn from the results obtained here:

(i) There is a significant increase in the pressure with ‘n’ and $\overline{U}$.

(ii) The sliding temperature is higher than that of pure rolling.

(iii) The effect of temperature is to reduce the load carrying capacity of the system.

(iv) The load and traction increases with ‘n’ and $\overline{U}$.

(v) The traction at the lower surface is higher than that of the upper surface because of the more speed of the lower surface.

(vi) The velocity of the lower surface being more that of the upper surface resulted the delta profile is up above the x- axis; otherwise it would have been the opposite to this.

(vii) There is also a significant increase in the mean film temperature with ‘n’ and $\overline{U}$ both; hence it is imperative to treat the consistency $\overline{m}$ of the power law fluid to vary with pressure & temperature.

(viii) The effect of the peclect number ($\overline{P}_e$) is more in the inlet region.

REFERENCES

1. Li-Ming Chu, H-C. Hsu, J-R. Lin and Yuh-Ping Chang, Tribol. Int., **42**, 1154 (2009).

2. D. Dowson and A. V. Whitaker, Am. Soc. Lub. Eng., **8**, 224-234 (1965).
3. J. Wang, Shiyue Qu and Peiran Yang, Tribol. Int., **34**, 191-202 (2001).
4. N. Anandan, R. K. Pandey, C. R. Jagga and M. K. Ghosh, J. Eng. Trib., **220**, 535-547 (2006).
5. P. Kumar, S. C. Jain and S. Ray, Tribol. Int., **41**, 482-492 (2008).
6. H. Sojoudi and M. M. Khonsari, J. Trib., **132**, 012101-1 to 012101-9 (2010).
7. B. Vengudusamy, A. Grafl, F. Novotny-Farkas and W. Schofmann, Lub. Sci., **26**, 229-249 (2014).
8. J. A. Khaled and A. E. Abdallah, Trib. Int., **41**, 1237-1246 (2008).
9. E. Durak, O. Salmon and C. Kurbanoglu, Ind. Lub. Trib., **60(6)**, 309-316 (2008).
10. P. Sinha and C. Singh, J. Lub. Techn., **104**, 168-172 (1982).
11. P. C. Mishra, Trib. Ind., **36(2)**, 211-219 (2014).
12. D. Prasad, P. Singh and P. Sinha, J. Trib., **110**, 653-658 (1988).
13. D. Prasad, J. B. Shukla, P. Singh, P. Sinha and R. P. Chhabra, Trib. Int, **24**, 239-246 (1991).
14. D. Prasad, P. Singh and Prawal Sinha, J. Trib., **115**, 319-326 (1993).
15. D. Prasad and S. V. Subrahmanyam, Proc. Intl. Conf. Adv. Tribol. Eng. Sys., Springer India (2014) pp. 127-141.
16. A. Almqvist, J. Fabricius, R. Larsson and P. Wall, J. Trib., **136**, 011706-1 to 011706-7 (2014).
17. Dong Zhu, J. Tribol. **133**, 041001-041014 (2011).
18. P. Kumar and M. M. Khonsari, J. Tribol., **130**, 041505-1 to 041505-13 (2008).
19. F. Sadeghi, T. A. Dow and R. R. Johnson, J. Tribol, **109**, 519-524 (1998).
20. S. H. Wang, D. Y. Hua and H. H. Zang, J. Tribol., **110**, 583-586 (1998).
21. J. Y. Jang and M. M. Khonsari, J. Tribol., **132**, 034501-1 to 034501-6 (2010).
22. D. Prasad, P. Singh and P. Sinha, Wear, **119**, 175-190 (1987).
23. L. Rong-Tsong and B. J. Hamrock, J. Tribol., **111**, 1-7 (1989).

24. V. S. Sajja and D. Prasad, Ind. Lub. Trib., **67(3)**, 246-255 (2015).

25. P. K. Saini, P. Kumar and P. Tandon, J. Eng. Trib., **221**, 141-153 (2007).

26. M. K. Ghosh and B. J. Hamrock, ASLE, **28**, 159-171 (1985).

27. F. Sadeghi and T. A. Dow, J. Trib., **109**, 512-518 (1987).

28. D. Prasad, S. V. Subrahmanyam and S. S. Panda, Int. J. Eng. Sci. Adv. Techn., **2**, 422-437 (2012).

29. V. Bruyere, N. Fillot, G. E. Moreles-Espejel and P. Vergne, Trib. Intl, **46**, 3-13, (2012).

30. P. Kumar and M. M. Khonsari, Trib. Int., **42**, 1522-1530 (2009).

31. G. E. Morales-Espejel and A. W. Wemekamp, J. Eng. Tribol., **221**, 15-34 (2008).

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