Int. J. Chem. Sci.: 14(3), 2016, 1633-1640
ISSN 0972-768X
www.sadgurupublications.com

STRUCTURAL ANALYSIS OF ELLIPTICAL PRESSURE VESSELS WITH CIRCULAR CROSS SECTION

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ABSTRACT

This paper shows the analysis of elliptical pressure vessels with circular cross section subjected to internal pressure. Pressure vessels have a very wide range of applications in the propulsion industry and work under very high internal pressures and hence these vessels must be mechanically stable to store the propellant under the required pressure conditions. Therefore, proper analysis of these vessels must be conducted before they can be sent for machining processes. This paper shows analytical solutions as well as Finite Element Solutions to analyse the pressure vessels. A 3D model of a pressure vessel is also created and simulated under required constraints and pressure loads and solved using computational finite element solver. In the end, the results are compared and an attempt is made to restrict the relative error under 5%.

Key words: Elliptical pressure vessels, Circular cross section.

Nomenclature

Symbol	Physical quantity	Unit
σ_l	Longitudinal stress	MPa
σ_c	Circumferential stress	MPa
F_l	Longitudinal force	N
F_c	Circumferential force	N
p	Pressure	MPa
r_1	Radius of curvature in circumferential direction	mm
r_1	Radius of curvature in longitudinal direction	mm

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INTRODUCTION

Pressure vessels are extensively used in aerospace and manufacturing industries. The design of the encapsulated pressure vessels should be good enough to bear the internal pressure and stresses given by the fluid kept in it. This paper is regarding the finite element analysis and stress equations on the surface of pressure vessels. The burst pressure is obtained by the data sheet and we have to calculate the hoop stress induced in the vessel. Our main is aim to successfully design the vessel keeping all safety factors such as hoop stress into consideration. The design of the vessel is modeled and studied. The vessels are generally subjected to high external and internal pressures so while calculating the equations for stresses we need to see all the forces acting on the vessel. There are various designs of pressure vessels out of which we have studied for the ellipsoidal tank with circular cross section. The material needed to design the tank is titanium which is extensively used in aerospace industries. Some industries such as oil industries use tanks for storage purposes for various flammable or inflammable fluids, therefore, they require their tank specifications to match their criteria.

Analytical solution

This section provides the governing analytical equations to solve membrane stresses for axis-symmetric pressure vessels with circular cross sections. Consider the section of the pressure vessel as shown:

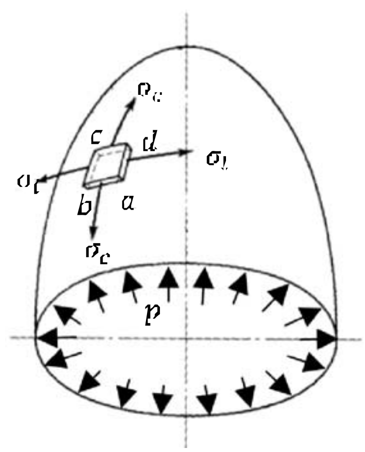

Fig. 1[1]

The vessel is subjected to a pressure p as shown. Consider a small element $abcd$ with dimensions $ds_1 \times ds_2$ as shown. Note that for simplicity the element chosen is oriented along longitudinal and circumferential directions of the section, so that forces only in the normal directions act on the element. Now studying the curved element ds_1 and ds_2 separately, we have:

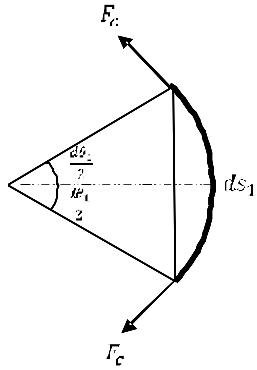

Fig. 2

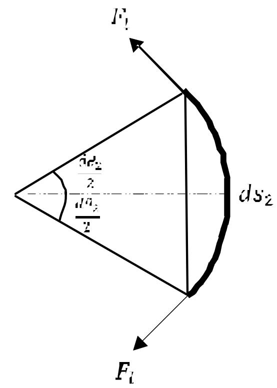

Fig. 3

Now, the resultant circumferential force is given by:

$$F_c = \sigma_l \times t \times ds_2 \qquad \ldots(1)$$

Also, $$F_1 = F_c \sin\frac{d\theta_1}{2} \approx F_c \frac{d\theta_1}{2} \qquad \ldots(2)$$

And the longitudinal force is given by:

$$F_l = \sigma_c \times t \times ds_1 \qquad \ldots(3)$$

and $$F_2 = F_l \sin\frac{d\theta_2}{2} \approx F_c \frac{d\theta_2}{2} \qquad \ldots(4)$$

Now the forces,

$$F_1 = \sigma_l \times t \times ds_2 \times \frac{d\theta_1}{2}$$

and $$F_2 = \sigma_c \times t \times ds_1 \times \frac{d\theta_2}{2}$$

But $ds_1 = r_1 d\theta_1$ and $ds_2 = r_2 d\theta_2$

$$\therefore F_1 = \sigma_l \times t \times r_2 d\theta_2 \times \frac{d\theta_1}{2}$$

and $$F_2 = \sigma_c \times t \times r_1 d\theta_1 \times \frac{d\theta_2}{2}$$

In order to balance these forces, a force is arisen due to the internal pressure .

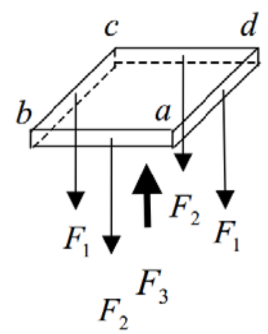

Fig. 4

Where

For static equilibrium,

Which on solving gives,

$$\frac{\sigma_l}{r_1}+\frac{\sigma_c}{r_2}=\frac{p}{t} \qquad \ldots(5)$$

For an ellipsoidal curvature, where the propellant is free from end restraints, no longitudinal stress is induced in the surface. There the equation for circumferential stress may be given by:

$$_c = \frac{pb}{t} \qquad \ldots(6)$$

Where Semi-major axis of the ellipse

The finite element method

In solving a problem, which involves deformation of linear elastic solids, the first method used is the study of strength of materials. It involves 3 basic types of field equations – equilibrium, constitutive (stress-strain) and compatibility (strain-displacement) and solving for the unknowns after appropriate substitution has been made. The second method is the Energy method, which equates the state of equilibrium to the condition of minimum potential energy. It is the base of the solution criteria of FEM.

Mathematically, the energy principle states that:

$$\Pi = U - W \quad \ldots(7)$$

Where, $\Pi =$ Potential Energy, $U =$ Internal Energy, $W =$ External work done on the system

In FEM, we seek the condition where $\delta\Pi = 0$.

The Ritz method

In order to obtain $\delta\Pi = 0$, classical methods can be used to solve the differential equations but they can lead to tedious and non-solvable conditions. Under such circumstances, numerical methods come into play, which are approximate methods of solving a differential equation and then iterating the solution a number of times to reach closer to the correct solution. And since FEA relies on computers in practice, it is advisable to use approximate methods. One such method is the Ritz method, which involves estimating the form of the solution for U (assuming that an appropriate expression of the equation __ as a function of U has been first developed). The general form of this estimate has been expressed as:

$$U(x) = \sum_r a_r \Psi_r(x) \quad \ldots(8)$$

Where $\Psi_r(x)$ is a function that must satisfy the boundary conditions of the problem and a_r are the unknown co-efficients. Next the partial derivatives of Π are taken with respect to each of the co-efficients.

$$\frac{\partial\Pi}{\partial a_1} = 0, \frac{\partial\Pi}{\partial a_2} = 0, \ldots, \frac{\partial\Pi}{\partial a_r} = 0 \quad \ldots(9)$$

Thus, a system of simultaneous algebraic equations is setup which can be solved for a_r to obtain the value of the displacement and subsequently the value of stress.

In FEM we use the same concept only the solid body is discretised into infinitesimally small elements which are interconnected by nodes. The nodal displacement is then calculated using numerical methods as shown:

$$\frac{\partial\Pi}{\partial u_1} = 0, \frac{\partial\Pi}{\partial u_2} = 0, \ldots, \frac{\partial\Pi}{\partial u_k} = 0 \quad \ldots(10)$$

The finite element model of the shell was created by splitting the vessel into half which reduces the computational time leaving no effects in the result obtained as the circumferential stress is same throughout.

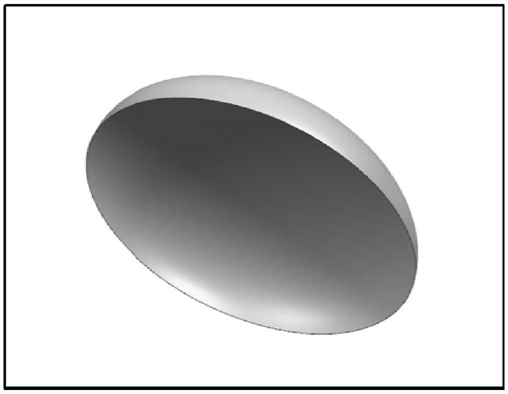

Fig. 5: Vessel split into half

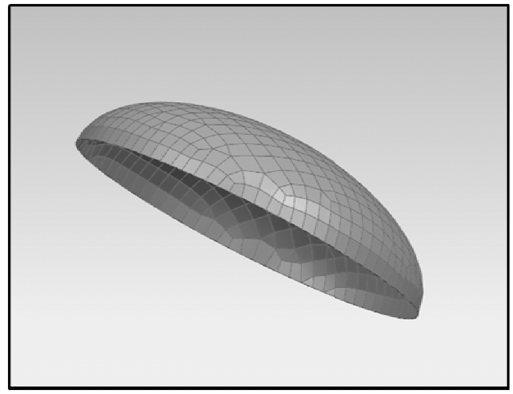

Fig. 6: Finite element meshed model of the thin shell

The shell with 2 mm thickness was analyzed under 20 MPa of internal pressure with the circular cross section fixed. On solving the above Finite Element Model, we obtain the following nodal displacement and elemental stress for the nodes stretching from one point on the circumference to the point diametrically opposite to it. The material chosen was Titanium.

RESULTS AND DISCUSSION

Due to the application of internal pressure of 20 MPa, the shell surface undergoes deflection. The nodal displacement result using FEA is shown along with the deflection of each node below:

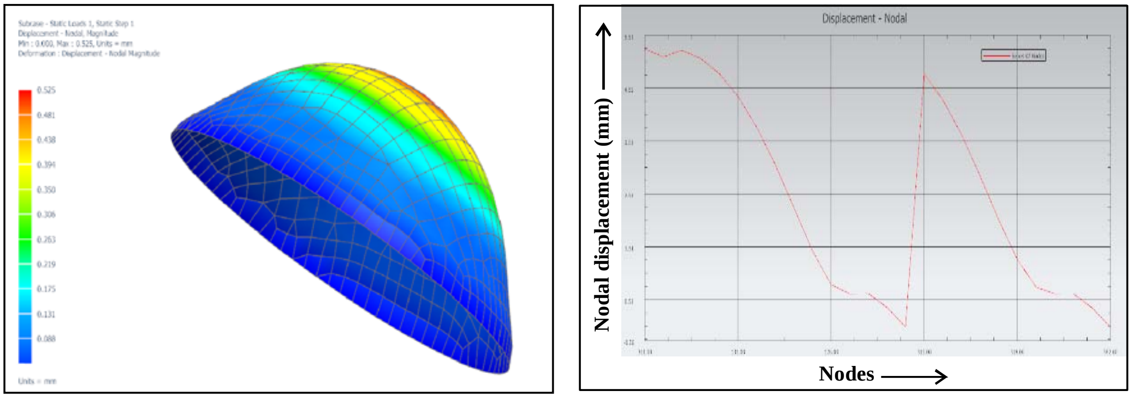

Fig. 7: Nodal displacement results

Due to this internal pressure, a stress is developed along the shell membrane as discussed in the theory above and the elemental stress result using FEA is shown below with stress concentration at various nodes:

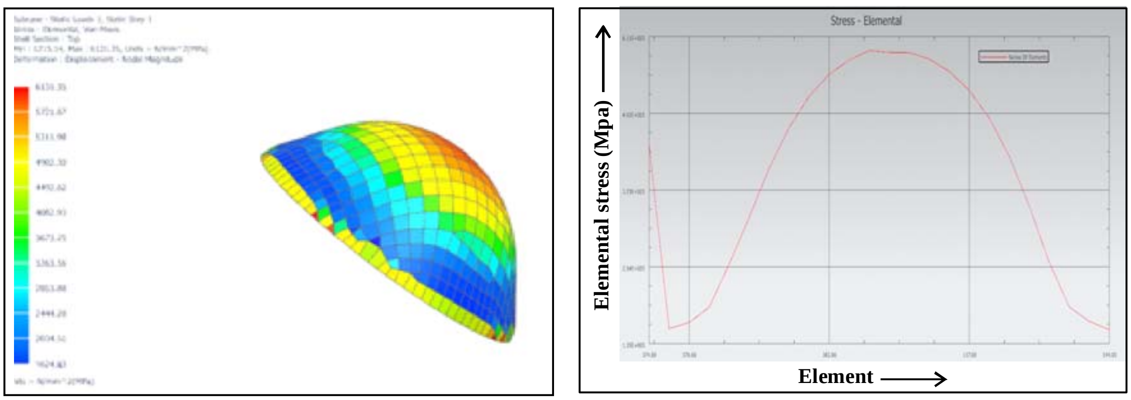

Fig. 8 Elemental stress results

The results above show the maximum displacement to be 0.525 mm and maximum elemental stress to be 613 MPa. The yield strength of Titanium is 804 MPa and hence, the maximum stress induced under 20 MPa is much below the yield strength. The study shows that the considered thickness for the given shell is optimum for the shell to prevent failure of the shell.

CONCLUSION

The paper describes a study on the mechanical behaviour of thin elliptical shells with a circular cross-section on the application of internal pressure. The values chosen for internal pressure were taken from small satellite propulsion systems with pressure fed fuel injection. This however has no relevance to any commercial tanks available so far and the study is in progress to study the behaviour of axis-symmetric tanks and optimizing the design and make future plans for integrating pressure vessels.

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Revised : 16.06.2016 *Accepted : 19.06.2016*