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STEADY - STATE SELF–FOCUSING OF GAUSSIAN RIPPLED ELECTROMAGNETIC BEAMS IN PLASMAS: RELATIVISTIC NONLINEARITY

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ABSTRACT

This paper presents an investigation into the growth of a radially symmetrical spike, superimposed on a Gaussian laser beam propagating in unmagnetized plasma. Here we consider a collisionless plasma, where nonlinearity arising through the relativistic electron ponderomotive force in addition to the relativistic increase in mass.. The density depression is due to transverse pondromotive force on the electrons is larger than on the ions by the mass ratio. The electrons forced out of the radiation beam region set up an electrostatic restoring force which, on a slower time scale, causes the ions to be expelled. This density depression creates a local increase in the effective index of refraction and acts as an optical guide for the radiation beam. In addition to this self-focusing mechanism, a further reduction in the plasma frequency occurs in regions of high field intensity due to the relativistic mass increase of the electrons in the presence of the radiation beam. The small radius spike on the axis of the main beam grows very rapidly with the distance of propagation as compared to the self-focusing of the main beam. At higher intensities, the saturation effects of nonlinearity become predominant, making the nonlinear refractive index in the paraxial region have slower **r** dependence, and thus, letting the spike attract relatively less energy from its neighborhood.

Key words: Plasma, Guassian rippled, Self-focusing, Steady state, Electromagnetic beam, Relativistic nonlinearity

INTRODUCTION

There has been considerable interest in the interaction of electromagnetic radiations with plasmas. Such interactions have assumed importance on account of their relevance to the controlled thermonuclear fusion[1]. Besides, it has also importance in modern

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communication network as there we have interaction of radio wave / micro waves with the ionosphere. Among the nonlinear optical effects, the self-interaction of powerful laser beam occupies an important place. The interaction of intense laser beam with plasma modifies the dielectric constant of the medium. Moreover, the refractive index of the medium becomes intensity dependent[2] which further affects the propagation characteristics of the beam. Laser beam propagating in plasma can create its own waveguide in which geometric and diffraction divergence are removed and beam is self-focused. The phenomena of self-focusing and filamentation of laser beams in plasmas are closely related to each other[3]. Self focusing is the tendency of a laser beam (of definite width) to reduce its own transverse spatial dimension through interaction with the plasma medium. Filamentation, on the other hand, is the plane-wave analogue of this phenomenon and results in a spatially periodic distribution of the energy of the incident (uniform) plane wave. Most of the theoretical investigations of the self-focusing of laser beams in nonlinear media have been confined to cylindrical beams with a Gaussian intensity profile. However, direct and indirect experimental evidence reveals that an apparently Gaussian laser beam has intensity spikes that may lead to distortion of self –focusing in nonlinear media[4-6]. Most of the studies of filamentation instability have been carried out, when the main laser beam has uniform illumination[7-9]. However, energy exchange between the ripple and the main beam needs a little more serious consideration. The speckled intensity pattern of the beam leads to some important effects, which can significantly alter the propagation and energy deposition characteristic of the incident laser beam. Even though the large-scale intensity fluctuations are removed, the probability distribution of speckles produces a statistically significant number of highly intense speckles or hot spots. These intense localizations of laser intensity may initiate laser beam instabilities such as stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS), and filamentation[10-15]. More recently, growth of a spike on a Gaussian laser beam in collisionless and collisional plasma and dielectrics have been described by a number of authors[16-18]. Studies of laser focusing have revealed that the spike exchanges energy from main beam leading to a rapid growth of the spike. In collisionless plasma, relativistic nonlinearity exhibits a major effect on the nonlinear dynamics of the propagation of intense electromagnetic waves; hence, it would be worthwhile to examine its consequence on the growth of the spike.

In this paper we investigate the growth of a Gaussian spike of small radius on a high power Gaussian laser beam in a plasma properly accounting for the transfer of energy from the main beam to ripple. Here, we consider a collisionless plasma, where nonlinearity arising through the relativistic electron ponderomotive force in addition to the relativistic increase in mass. The propagation of an intense laser beam in collisionless homogeneous plasma can result in self-focusing by creating a density depression in the plasma as well as by increasing the electron mass by relativistic effects[8]. The density depression is due to transverse ponderomotive forces, which tend to expel plasma from high field regions. This density depression creates a local increase in the effective index of refraction and acts as an optical guide for the radiation beam. In addition to this self-focusing mechanism, a further reduction in the plasma frequency occurs in regions of high field intensity due to relativistic mass increase of the electrons in the presence of the laser beam. The self-focusing due to the density depression occurs on a longer time scale than does the relativistic mass increase self-focusing effect. Some general equations for self-focusing of laser beam in a nonlinear medium are presented and coupled equations for the beam width parameter of the Gaussian beam are derived; the width of the spike b, and the amplification parameter α of the spike for saturating nonlinearity profile. The equations are solved numerically to study the evolution of the spike with the distance of propagation. A discussion of results is given.

Propagation of a smooth Gaussian profile laser beam

Consider the propagation of a cylindrically symmetric Gaussian laser beam in uniform, static and collisionless plasma. A typical ansatz might be to assume the beam profile is Gaussian and that a Gaussian profile is maintained everywhere along the length of the beam. This is called the aberrationless approximation. For instance, at z = 0 the intensity distribution of the beam is given by

$$EE^*|_{z=o} = A^2_{oo}\ \exp\left(-\frac{r^2}{r_o^{\ 2}}\right) \qquad \ldots(1)$$

where $E = A\,(r, z)\exp\left(i\,(\omega t - kz)\right)$, $k = \dfrac{\omega}{c}\left(1-\dfrac{\omega_{P0}^2}{\omega^2}\right)^{\frac{1}{2}}$ and ω_{P0} is the unperturbed plasma frequency. The behavior of complex amplitude A is governed by the parabolic equation (2)

$$2\,ik\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\omega^2}{c^2}\phi\ (EE^*)\ A \qquad \ldots(2)$$

Where,

$$\phi\left(EE^*\right)=\frac{\omega_{po}^2}{\omega^2}\left(1-\frac{1}{\gamma_o}\right)-\frac{c^2}{\omega^2}\frac{\nabla^2\gamma_o}{\gamma_o},\ \omega_{po}=\left(\frac{4\pi n_o e^2}{m_o}\right)^{\frac{1}{2}},\ \gamma_o(r,\ z)=\left(1+a^2(r,z)\right)^{\frac{1}{2}}$$

is the relativistic mass factor and $a^2(r, z) = I\ (r, z) = \frac{e^2 EE^*}{m^2\omega^2 c^2}$ is the normalized laser field intensity. A radiation field a^2 (r, z) peaked on axis at r = 0 will produce an index of refraction profile peaked on axis. The term $\frac{\omega_{po}^2}{\omega^2}\left(1-\frac{1}{\gamma_o}\right)$ is a consequence of the relativistic increase in mass, while the term $\frac{c^2}{\omega^2}\ \frac{\nabla^2\gamma_0}{\gamma_o}$ stems from the nonlinear ponderomotive force on the electrons. Both these terms determine the nonlinear refraction force and promote self-focusing of the beam. In the absence of any spikes, we may express A as

$$A. = A_0 \exp(-ikS), \qquad \dots(3)$$

where the real quantities A_0 and S (the eikonal of the wave) are both functions of r and z. Substituting for A in Eq. (2) we obtain the following two equations of A_0 and S in a cylindrical coordinate system.

$$2\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial r}\right)^2=\phi\left(A_0^2\right)+\frac{c^2}{\omega^2 A_0}\nabla_\perp^2 A_0 \qquad \dots(4)$$

$$\frac{\partial A_0^2}{\partial z}+\left(\frac{\partial S}{\partial r}\right)\frac{\partial A_0^2}{\partial r}+A_0^2\nabla_\perp^2 S=0 \qquad \dots(5)$$

where $\nabla_\perp^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}$. It is important to mention here that the Gaussian distribution of intensity is realized when the laser is operating in the TEM_{00} mode and, indeed, most of the lasers do operate in this mode. Hence, Eq. (4) is applicable to a large number of experimental situations. Following Akhmanov et al.[13], we express A_0^2 and S, in paraxial ray approximation, as

$$A_0^2(r,z)=\frac{A_{00}^2}{f^2}\exp\left(-r^2/r_0^2 f^2\right) \qquad \dots(6)$$

$$S(r,z)=\frac{r^2}{2}\beta(z)+\Psi(z),\ \beta=\frac{1}{f}\frac{df}{dz}$$

where f is the ratio of the beam diameter to its value at z = 0, β corresponds to the inverse radius of the curvature of the wave front and $a' = r_0 f$ is the width of the main beam in the medium. The beam width parameter, f (z), scales the beam radius. As f (z) decreases, the intensity increases as $1/f^2(z)$ in order to conserve energy. The eikonal gives an ordinary differential equation for f(z), if it is expanded to order r^2. This expansion is known as the paraxial ray approximation since it emphasizes the importance of the paraxial (those near r = 0) rays. The aberrationless and paraxial ray approximations are essentially synonymous since they yield a set of solutions characterized by a single parameter that scales the shape of the beam[3]. Using Eq. (6) in Eqs. (4) and (5), we obtain equation for the beam width parameter f :

$$\frac{d^2 f}{dz^2} = \frac{1}{R_d^2 f^3} - \frac{A_{00}^2}{r_0^2 f^3}\phi'\left(\frac{A_{00}^2}{f^2}\right) \qquad \ldots(7)$$

where ϕ' is the derivative of ϕ with respect to its arguments and terms of order higher than r^2 have been neglected. Eq. (7) is difficult to solve analytically and thus, it has been solved numerically by the computer with appropriate boundary conditions. We may take f =1 and $\frac{df}{dz} = 0$ corresponding to an initially plane wave front.

Treatment for Gaussian perturbation

In the presence of a single spike of Gaussian profile propagating coaxially with the main electromagnetic beam, we may express A as-

$$A = (A_0 + A_1)\exp(-ikS), \qquad \ldots(8)$$

where $A_1 = A_{1r} + iA_{1i}$ is the complex electric field of the perturbation with A_{1r}, $A_{1i} << A_o$, A_o being the unperturbed amplitude of the main beam and

$$\phi\left(A^2\right) \approx \phi\left(A_0^2\right) + \frac{\partial\phi}{\partial A_0^2} 2A_0 A_1 \qquad \ldots(9)$$

Using Eqs. (8) and (9) in Eq. (2) and employing Eqs. (4) and (5), we obtain two coupled equations for A_{1r} and A_{1i} in the paraxial ray approximation as-

$$2k\frac{\partial A_{1r}}{\partial z} + \nabla_\perp^2 A_{1i} + 2kr\beta\frac{\partial A_{1r}}{\partial r} + 2K\beta A_{1r} + \frac{1}{r_0^2 f^2}\left(2 - \frac{r^2}{r_0^2 f^2}\right)A_{1i} = 0, \qquad \ldots(10)$$

$$2k\frac{\partial A_{1i}}{\partial z}-\frac{1}{r_0^2f^2}\left(2-\frac{r^2}{r_0^2f^2}\right)A_{1r}-\nabla_\perp^2A_{1r}+2kr\beta\frac{\partial A_{1i}}{\partial r}+2k\beta A_{1i}-\frac{2\omega^2}{c^2}A_0^2\frac{\partial\phi}{\partial A_0^2}A_{ir}=0 \quad \ldots(11)$$

We take the forms of A_o and S given by Eqs. (6) and A_{1r} , A_{1i} as-

$$A_{1r},\ A_{1i}=a_o,\ a_1\exp\left[\alpha(z)\right]\exp\left(-\frac{r^2}{2b^2(z)}\right) \quad \ldots(12)$$

Where $\alpha(z)$ is the amplification parameter and b (z) is the spike width. Substituting for A_{1r} , A_{1i} in Eqs. (10) and (11) and collecting r and r^2 terms independently on both sides, we get two pairs of coupled equations for a_o and a_1-

$$\left(2k\frac{d\alpha}{dz}+2k\beta\right)a_o=\left(\frac{2}{b^2}-\frac{2}{r_0^2f^2}\right)a_1, \quad \ldots(13)$$

$$\left(2k\frac{d\alpha}{dz}+2k\beta\right)a_1=\left(-\frac{2}{b^2}+\frac{2}{r_0^2f^2}+2\frac{\omega^2}{c^2}\left(A_0^2\frac{\partial\phi}{\partial A_0^2}\right)_{r=0}\right)a_0, \quad \ldots(14)$$

$$\left(\frac{2k}{b^3}\frac{db}{dz}-2k\beta\right)a_0=\left(-\frac{1}{b^4}+\frac{1}{r_0^2f^4}\right)a_1 \quad \ldots(15)$$

$$\left(\frac{2k}{b^3}\frac{db}{bz}-\frac{2k\beta}{b^2}\right)a_1=\left[\frac{1}{b^4}-\frac{1}{r_0^2f^4}+\frac{2\omega^2}{c^2}\left(\frac{\partial}{\partial r^2}\left(A_0^2\frac{\partial\phi}{\partial A_0^2}\right)_{r=o}\right)\right]a_0, \quad \ldots(16)$$

Multiplying Eq. (13) with Eq. (14) and Eq. (15) with (16), we obtain the following set of equations for α and b :

$$\frac{d\alpha}{dz}=\frac{1}{k}\left[\left(\frac{1}{b^2}-\frac{1}{r_o^2f^2}\right)\left\{\frac{\omega^2}{c^2}\left(A_o^2\frac{\partial\phi}{\partial A_o^2}\right)_{r=o}-\left(\frac{1}{b^2}-\frac{1}{r_o^2f^2}\right)\right\}\right]^{\frac{1}{2}}-\beta(z) \quad \ldots(17)$$

$$\frac{db}{dz}=\frac{b^3}{2k}\left[\left\{\left(\frac{1}{b^4}-\frac{1}{r_0^4f^4}\right)\left(-\frac{1}{b^4}+\frac{1}{r_0^4f^4}+\frac{2\omega^2}{c^2}\left(\frac{\partial}{\partial r^2}\left(A_0^2\frac{\partial\phi}{\partial A_0^2}\right)_{r=o}\right)\right)\right\}^{\frac{1}{2}}+\frac{2k\beta}{b^2}\right] \quad \ldots(18)$$

We solve these differential equations with initial conditions.

$$b\,(z=0)=b_o\,;\quad \alpha\,(z=0)=0 \qquad \ldots(19)$$

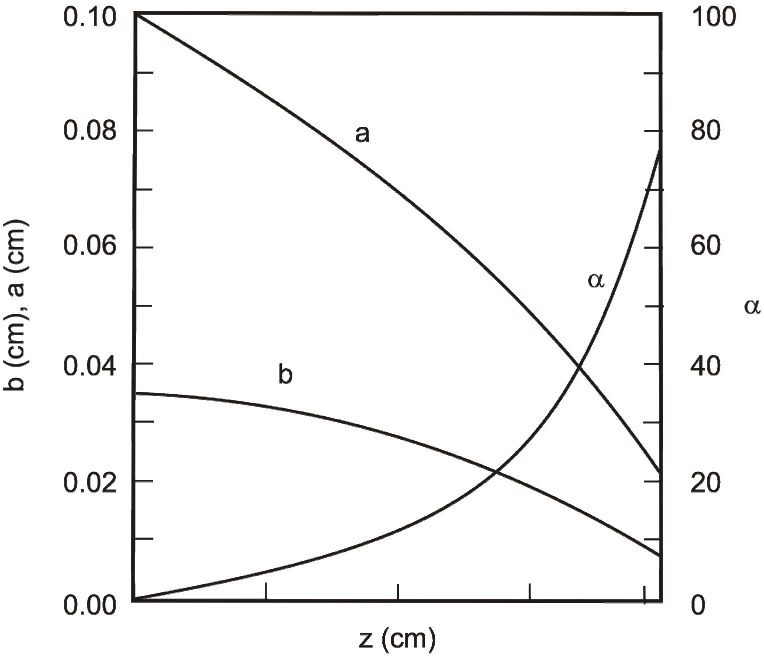

Fig. 1: Variation of beam width parameter a, spike width b and amplification parameter α with distance of propagation z (cm.) for $r_o = 0.1$ cm, $\omega^2{}_{po}/\omega^2 = 0.01$, $\omega = 9.4 \times 10^{13}\ s^{-1}$, a (0) = 0.14, $P_{crit} = 17 \times 10^{11}$W and b(z = 0) $\approx$ 0.033 cm.

One may define amplification length as $Z_A = \left(\frac{d\alpha}{dz}\right)^{-1}$. One may note from Eqs. (17) and (18) that α varies more rapidly with z than b varies with z and $\frac{d\alpha}{dz}$ has an extremum value for those values of b for which d (RHS) / db = 0, where RHS is the right hand side of Eq. (17). This gives the values of beam diameter b as-

$$\frac{1}{b^2} = \frac{1}{2}\frac{\omega^2}{c^2}\left(A_o^2\frac{\partial\phi}{\partial A_o^2}\right)_{r=o} + \frac{1}{r_o^2 f^2}\,. \qquad \ldots(20)$$

The corresponding value of $\frac{d\alpha}{dz}$ is-

$$\frac{d\alpha}{dz}=\frac{1}{2k}\frac{\omega^2}{c^2}\left(A_o^2\frac{\partial\phi}{\partial A_o^2}\right)_{r=o}-\beta(z) \qquad \ldots(21)$$

We have solved Eqs. (7), (17) and (18) numerically with boundary conditions f (z = 0) = 1, df/dz $|_{z=o}$ = 0, b (z = 0) = b_{opt} and α (z = 0) = 0 for the following set of parameters r_o = 0.1 cm., $\omega^2{}_{po}/\omega^2 = 0.01$, $\omega = 9.4\times10^{13}$ s^{-1}, a (0) = 0.14, P_{crit} = 17 x 10^{11}W. and b (z = 0) $\approx$ 0.033 cm. The results are plotted in Fig. 1.

For these parameters, a (r,z) acquires larger values as z increases; hence, saturating effects of nonlinearity are important. The nonlinearity in refractive index causes focusing of the main beam. For these parameters, the laser beam undergoes self focusing attaining f_{min} at z =17 cm. Beyond this point f increases. Initially the beam focuses due to nonlinear refraction due to relativistic ponderomotive effect and reduction in the plasma frequency occurs in regions of high field intensity due to the relativistic mass increase of the electrons in the presence of the intense beam. However, as the intensity in the axial region builds up the plasma is almost fully depleted from this region weakening the self-focusing effect. The diffraction effect becomes quite severe at this stage leading to divergence of the main beam. The spike width (b) decreases with distance of propagation. Eq. (21) gives the growth parameter for the fastest growing perturbation. Eq. (17) indicates that amplification parameter (α) increases with the distance of propagation; however, $\frac{df}{dz}$ decreases slowly as a result of the self focusing of the main beam.

DISCUSSION

An on-axis spike in the intensity distribution of a Gaussian laser beam grows rapidly as the beam propagates in plasma. The nonlinearity arises through the combined effect of relativistic electron ponderomotive force in addition to the relativistic increase in mass. For lower value of z, α increases linearly. For higher values of z, α increases asymptotically to infinity. Our theory gives a smooth matching between the exponential growth of perturbations in a linearized instability theory and the sharp self-focusing thresholds expected for smooth Gaussian profile electromagnetic beams propagating in nonlinear medium.

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