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RADIATION EFFECT WITH THERMAL MASS DIFFUSION IN MHD MIXED CONVECTION FLOW FROM A POROUS VERTICAL PLATE WITH OHMIC HEATING

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ABSTRACT

The effects of radiation on heat and mass transfer transfer in MHD mixed convection flow past an infinite vertical porous plate with Ohmic heating and viscous dissipation have been discussed. Solutions have been derived for the velocity, temperature field, and concentration profiles using multi-parameter perturbation technique. The results are analyzed graphically to observe the influence of various parameters like Schmidt number (Sc), Prandtl number (Pr), magnetic parameter (M), radiation parameter (F), and porosity (k).

Key words: Radiation, Magnetic field, Heat-transfer, Porosity, Viscous dissipation.

INTRODUCTION

In some industrial applications like glass production, space technology applications, cosmial flight aerodynamics rocket, plasma physics, etc., the radiation effects are found to be more significant. Magnetohydrodynamics flows has applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, and in the motion of Earths core. MHD free convection flows have significant applications on account of their varied importance. Heat and mass transfer on flow past a vertical plate have been studied by several authors; viz. Soundalgekar and Ganesan[1], Khair and Bejan[2] and Lin and Wu[3] in numerous ways to interpret various physical aspects. The effects of thermal radiation on free convective flow past a moving vertical plane was examined by Raptis and Perdikis[4]. Hussain and Takhar[5] investigated the radiation effects on mixed convection along an isothermal vertical plate. Muthucumaraswamy and Ganesan[6] have studied the radiation effects on flow past an impulsively started infinite vertical plate with variable temperature.

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Formulation of the problem

We consider the mixed convection flow of an incompressible and electrically fluid such that x^* -axis is taken along the plate in upward direction and y^*-axis. Since the motion is two dimensional and length of the plate is large, all the physical variables are independent of x^*. Let u^* and v^* be the components of velocity in x^* and y^* directions, respectively taken along and perpendicular to the plate. The governing equations of continuity, momentum and energy for a flow of an electrically conducting fluid along a hot, non-conducting porous vertical plate is given by –

$$\frac{\partial v^*}{\partial y^*} = 0 \text{ i.e. } v^* = -v_0 \text{ (constant)} \qquad \ldots(1)$$

$$\frac{\partial p^*}{\partial y^*} = 0 \Rightarrow p^* \text{ is independent of } y^* \qquad \ldots(2)$$

$$\rho v^* \frac{\partial u^*}{\partial y^*} = \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta \left(T^* - T_\infty\right) - \sigma B_0^2 u^* + \sigma g \beta^* \left(C - C_\infty\right) - \frac{\nu}{k} u^* \qquad \ldots(3)$$

$$\rho c_p v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa \partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{\partial q_r}{\partial y^*} + \sigma B_0^2 u^{*2} \qquad \ldots(4)$$

$$v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \qquad \ldots(5)$$

Here, g is the acceleration due to gravity, T^* the temperature of the fluid near the plate, T_∞ the free stream temperature, C^* concentration, β the coefficient of thermal expansion, K the thermal conductivity, p^* the pressure, Cp the specific heat of constant pressure, B_0 the magnetic field coefficient, μ viscosity of the fluid, qr^* the radiative heat flux, ρ the density, σ the magnetic permeability of fluid, V_0 constant suction velocity, ν the kinematic viscosity, D the molecular diffusivity, and k the porosity of the plate. The radiative heat flux is given by –

$$\frac{\partial q_r^*}{\partial y^*} = 4\left(T^* - T_\infty\right) I' \qquad \ldots(6)$$

Where $$I^{'} = \int_0^{\infty} K \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda ,$$

$k_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is plank's function. The boundary conditions are

$$y^* = 0 : u^* = 0, T^* = T_w , C_\infty = C \quad \ldots(7)$$

$$y^* \to \infty : u^* \to 0, T^* \to T_\infty , C^* \to C_\infty$$

Introducing the following non – dimensional quantities

$$y = \frac{v_0 y^*}{v},\ u = \frac{u^*}{v_0},\ M^2 = \frac{B_0{}^2 v^2 \sigma}{v_0{}^2 \mu},\ \Pr = \frac{\mu c_p}{\kappa},\ \theta = \frac{T^* - T_\infty}{T_w - T_\infty},\ C = \frac{C^* - C_\infty}{C - C_\infty},$$

$$k = \frac{k' V_0{}^2}{v^2} \quad \ldots(8)$$

$$E = \frac{v_0{}^2}{C_p \left(T_w - T_\infty\right)},\ Gr = \frac{\rho g \beta v^2 \left(T_w - T_\infty\right)}{v_0{}^3 \mu},\ Sc = \frac{v}{D},\ Gm = \frac{\rho g \beta^* \left(C - C_\infty\right) \mu}{\mu v_0{}^3}$$

and $$F = \frac{4 v I^{'}}{\rho C_p v_0{}^2}$$

The equations becomes

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} - M^2 u = -G_r \theta - G_m c - \frac{u}{k} \quad \ldots(9)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \Pr \frac{\partial \theta}{\partial y} - F \Pr \theta + \Pr E \left(\frac{\partial u}{\partial y}\right)^2 + \Pr E M^2 u^2 = 0 \quad \ldots(10)$$

$$\frac{\partial^2 c}{\partial y^2}+Sc\frac{\partial c}{\partial y}=0 \qquad \ldots(11)$$

Where Gr = Grashoff number, Pr = Prandtl number M = Magnetic parameter, F = Radiation parameter, Sc = Schmidt number, E = Eckret number, and k = Porosity.

The corresponding boundary conditions in dimensionless form are reduced to –

$$\begin{aligned} &y=0:u=0,\theta=0,C=1\\ &y\to\infty:u\to 0,\theta\to 0 \end{aligned} \qquad \ldots(12)$$

As the flow is incompressible due to the joules dissipation, we can assume –

$$u(y)=u_0(y)+Eu_1(y)+o\left(E^2\right)$$

$$\theta(y)=\theta_0(y)+E\theta_1(y)+o\left(E^2\right) \qquad \ldots(13)$$

$$C(y)=C_0(y)+EC_1(y)+o\left(E^2\right)$$

$$u_0''+u_0'-\left(M^2-\frac{1}{k}\right)u_0=-G_r\theta_0-G_m c_0 \qquad \ldots(14)$$

$$u_1''+u_1'-\left(M^2-\frac{1}{k}\right)u_1=-G_r\theta_1-G_m c_1 \qquad \ldots(15)$$

$$\theta_0''+\Pr\theta_0'-F\Pr\theta_0=0 \qquad \ldots(16)$$

$$\theta_1''+\Pr\theta_1'-F\Pr\theta_1=-\left(\Pr u_0'^2+M^2u_0^2\right) \qquad \ldots(17)$$

$$c_0''+Sc\,c_0'=0 \qquad \ldots(18)$$

$$c_1''+Sc\,c_1'=0 \qquad \ldots(19)$$

And the corresponding boundary conditions are

$$y = 0 : u_0 = 0, \quad u_1 = 0, \theta_0 = 1, \quad C_0 = 1, \quad C_1 = 0$$

$$y \to \infty : u_0 \to 0, \quad u_1 \to 0, \ \ \theta_0 \to 0, \ \ \theta_1 \to 0, \ \ C_0 \to 0, \ C_1 \to 0 \qquad \ldots(20)$$

Solving equations (14) to (19) using (20)

$$u_0 = \left(B_1 + B_2\right) e^{A_4 y} - B_1 e^{A_2 y} - B_2 e^{-Scy}$$

$$u_1 = \alpha_2 e^{A_4 y} - \alpha_1 \frac{G_r}{B_3} e^{A_2 y} - \Pr\left[\frac{A_4^{\,2}\left(B_1 + B_2\right)^2}{A_5 B_4} e^{2A_4 y} - \frac{A_2^{\,2} B_1^{\,2}}{A_6 B_5} e^{2A_2 y}\right]$$

$$-\Pr M^2 \left[\begin{array}{l} \dfrac{B_1^{\,2}}{A_5 B_4} e^{2A_4 y} + \dfrac{B_1^{\,2}}{A_6 B_5} e^{2A_2 y} - \dfrac{2B_1^{\,2}}{A_{11} B_6} e^{A_8 y} + \dfrac{B_2}{A_5 B_4} e^{2A_4 y} + \dfrac{B_2^{\,2}}{A_7 B_7} e^{-2Scy} - \\ \dfrac{2B_2^{\,2}}{A_{12} B_8} e^{A_9 y} + \dfrac{2B_1 B_2}{A_5}\left(\dfrac{e^{2A_9 y}}{B_4} - \dfrac{e^{A_8 y}}{A_{11} B_{10}}\right) \end{array}\right]$$

$$\theta_0 = e^{A_2 y}$$

$$\theta_1 = \alpha_1 e^{A_2 y} - \Pr\left[\frac{A_4^{\,2}\left(B_1 + B_2\right)^2}{A_5} e^{2A_4 y} - \frac{A_2 B_1^{\,2}}{A_6} e^{2A_2 y} + \frac{B_2^{\,2} Sc^2}{A_7} e^{-2Scy}\right]$$

$$-\Pr M^2 \left\{\left[\frac{B_1^{\,2}}{A_5} e^{2A_4 y} + \frac{B_1^{\,2}}{A_6} e^{2A_2 y} - \frac{2B_1^{\,2}}{A_{11}} e^{A_8 y} \frac{B_2^{\,2}}{A_5} e^{2A_4 y} + \frac{B_2^{\,2}}{A_7} e^{-2Scy} - \frac{2B_2^{\,2}}{A_{12}} e^{A_9 y}\right]\right.$$

$$C_0 = e^{-Scy}$$

Where

$$A_1 = \frac{-\Pr + \sqrt{\Pr^2 + 4F\Pr}}{2}, \quad A_2 = \frac{-\Pr - \sqrt{\Pr^2 + 4F\Pr}}{2} \quad A_3 = \frac{-\left(1 - \sqrt{1 + 4\left(M^2 - \frac{1}{k}\right)}\right)}{2},$$

$$A_4 = \frac{-\left(1 + \sqrt{1 + 4\left(M^2 - \frac{1}{k}\right)}\right)}{2}$$

$$A_5 = 4A_4^2 + 2\Pr A_4 - F\Pr \quad A_6 = 4A_4^2 + 2\Pr A_2 - F\Pr \quad A7 = 4Sc^2 - 2\Pr Sc - F\Pr, \; A_8 = A_2 + A_4,$$

$$A_9 = A_4 - Sc,\; A_{10} = A_2 - Sc,\; A_{11} = A_8^2 + \Pr A_8 - F\Pr,\; A12 = A_9^2 + \Pr A_9 - F\Pr$$

$$A13 = A_{10}^2 + \Pr A_{10} - F\Pr$$

$$B_1 = \frac{G_r}{A_2^2 + A_2 - \left(M - \frac{1}{k}\right)^2},\; B_2 = \frac{G_m}{Sc^2 - Sc - \left(M - \frac{1}{k}\right)^2},\; B_3 = A_2^2 + A_2 - \left(M^2 - \frac{1}{k}\right),$$

$$B_4 = 4A_4^2 + 2A_4 - \left(M^2 - \frac{1}{k}\right),\; B_5 = 4A_2^2 + 2A_2 - \left(M^2 - \frac{1}{k}\right),\; B_6 = A_8^2 + A_8 - \left(M^2 - \frac{1}{k}\right),$$

$$B_7 = 4Sc^2 - 2Sc - \left(M^2 - \frac{1}{k}\right),\; B_8 = A_9^2 + A_9 - \left(M^2 - \frac{1}{k}\right),\; B_9 = A_9^2 - A_9 - \left(M^2 - \frac{1}{k}\right),$$

$$B_{10} = A_8^2 - A_8 - \left(M^2 - \frac{1}{k}\right),\; B_{11} = A_{10}^2 - A_{10} - \left(M^2 - \frac{1}{k}\right)$$

$$\alpha_1 = \Pr M^2\left[\frac{B_1^2}{A_5} + \frac{B_1^2}{A6} - \frac{2B_1^2}{A_{11}} + \frac{B_2^2}{A_5} + \frac{B_2^2}{A_7} - \frac{2B_2^2}{A12} + 2B_1B_2\left(\frac{11}{A_5} - \frac{}{A_{11}}\right)\right]$$

$$-\Pr\left[\frac{A_4^2 (B_1 + B_2)^2}{A_5} - \frac{A_2 B_1^2}{A_6} + \frac{B_2^2 Sc^2}{A_7}\right]$$

$$\alpha_2 = \alpha_1 \frac{G_r}{B_3} + \Pr\left[\frac{A_4^2 (B_1 + B_2)^2}{A_5 B_4} - \frac{A_2^2 B_1^2}{A_6 B_5}\right]$$

$$+\Pr M^2\left[\frac{B_1^2}{A_5B_4} + \frac{B_1^2}{A6B_5} - \frac{2B_1^2}{A_{11}B_6} + \frac{B_2^2}{A_5B_4} + \frac{B_2^2}{A_7B_7} - \frac{2B_2^2}{A_{12}B_8} + 2B_1B_2\left(\frac{1}{A_5B_4} - \frac{1}{A_{11}B_{10}}\right)\right]$$

RESULTS AND DISCUSSION

Fig. 1 and Fig. 2 depicts that increase of radiation parameter, porosity, results in decrease of velocity. For constant value of radiation parameter, porosity velocity increases near the plate and then decrease as we move far away from the plate. But Fig. 3 shows that increase of Schmidt number leads to increase of velocity.

(i) From Fig. 4, we can observe that increase of magnetic parameter results in decrease of velocity near the plate. Actually, a backward flow is observed. As we move far away from the plate, the velocity increases.

(ii) Fig. 5 exhibits that increase of Prandtl number leads to decrease in velocity near the plate and a reverse trend is observed as we move far away from the plate.

(iii) From Fig. 6, we can observe that increase of Schmidt number results in decrease of species concentration.

(iv) Fig. 7 and Fig. 8 shows that increase of radiation parameter and Prandtl number leads to decrease in temperature.

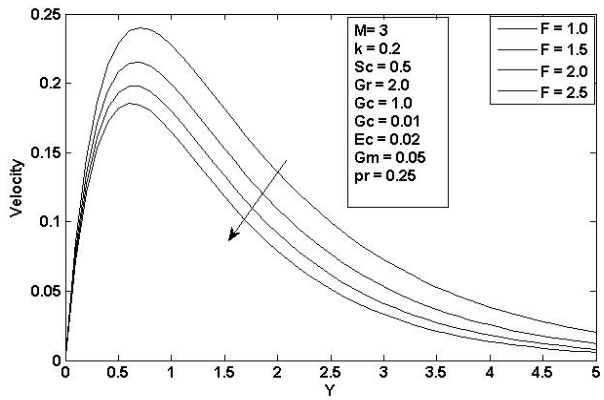

Fig. 1: Effect of radiation parameter on velocity

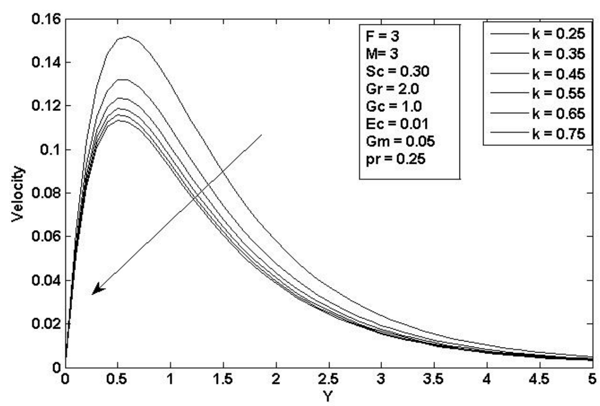

Fig. 2: Influence of porosity on velocoity

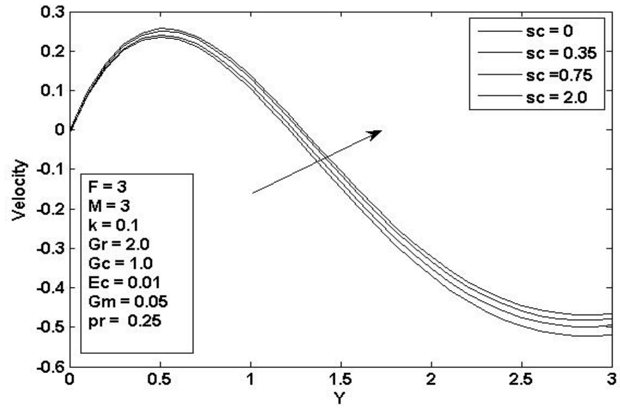

Fig. 3: Contribution of Schmidt number on velocity profiles

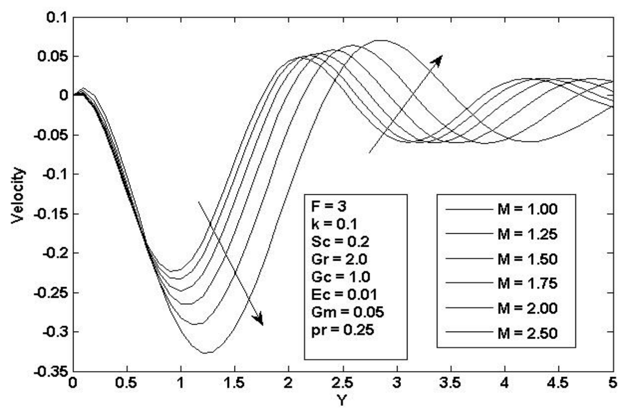

Fig. 4: Influence of magnetic parameter on velocity

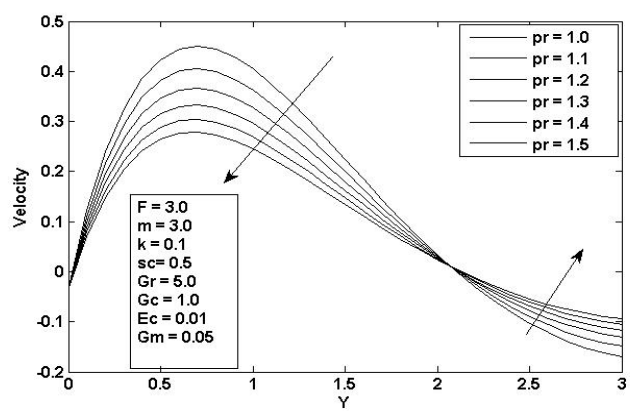

Fig. 5: Prandtl number effect on velocity

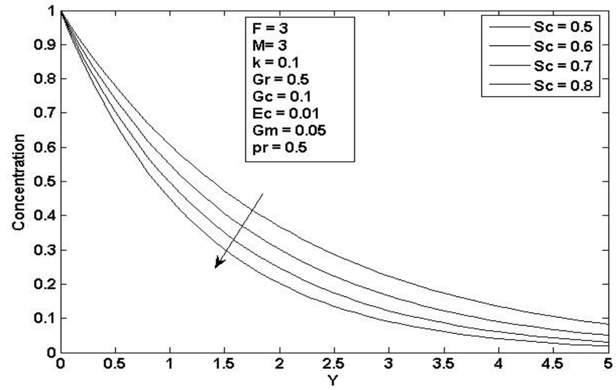

Fig. 6: Schmidt number effect on species concentration

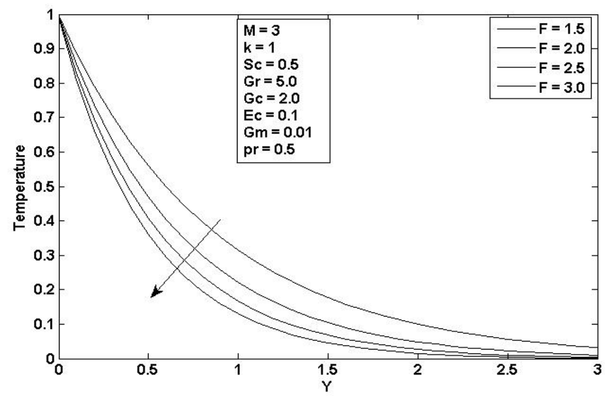

Fig. 7: Influence of radiation parameter on temperature

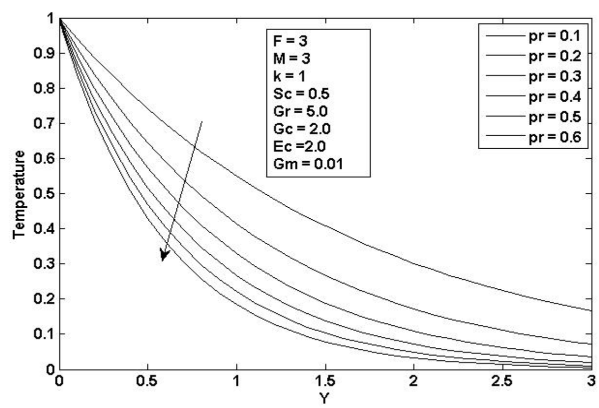

Fig. 8: Contribution of Prandtl number on temperature

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