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OBSERVABILITY OF FUZZY DIFFERENCE CONTROL SYSTEMS

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ABSTRACT

In this paper, we provide a way to incorporate difference equations with fuzzy sets to form a new fuzzy logic system called fuzzy difference control system which can be regarded as a new approach to intelligent control. One of its features, observability of this system will be studied. We obtain the sufficient condition for the system to be observable.

Key words: Fuzzy difference control system, Observability, Fuzzy difference equations, Fuzzy sets.

INTRODUCTION

The theory of difference equations plays an important role in areas such as digital control, digital filter design and image processing with the emergence of digital signal processing technology. The importance of control theory in applied mathematics and its occurrence in several fields such as mechanics, electromagnetic theory, thermodynamics and artificial satellites are well known. Observability is one of the fundamental concepts in modern mathematical control theory. The problem of observability for a system of ordinary differential equations was studied in[1,2] and for matrix Lyapunov systems by Murty et al.[3] Ding et al.[4] provide a way to combine differential equations with fuzzy sets to form a fuzzy logic systems called fuzzy dynamical systems and also studied observability of this system. Murty et al.[5-7] extended this approach to fuzzy dynamical matrix Lyapunov systems. We consider the linear difference control system of the form.

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$$x(n+1) = A(n)x(n) + B(n)u(n), \quad \mathrm{x}(0) = \mathrm{x}_0 \qquad \ldots(1.1)$$

$$y(n) = C(n)x(n) + D(n)u(n), \qquad \ldots(1.2)$$

where $A(n)$ is a non-singular matrix, B, C, D are matrix functions of n on $J = [0, L] \bigcap N,\ L \in N = \{0, 1, 2, ...\}$. If the inputs $u(n)$ are crisp, then it is the deterministic difference control system. Here, we take the input $u(n)$ is a fuzzy set on $\boldsymbol{R}^m$. We will prove that the state of the system (1.1), (1.2) is also a fuzzy set. The concept of observability is concerned with identification of the initial state by observing the output and input over a finite time. First, we introduce the notion of likely observability and obtain a sufficient condition for the observability of this system.

Preliminaries

In this section, we present some results on fuzzy sets and fuzzy difference equations which are useful for later discussion. Let $P_k(\boldsymbol{R^m})$ denotes the space of all nonempty compact convex subsets of $\boldsymbol{R}^m$. For any $A, B \in P_k(\boldsymbol{R^m})$, the Hausdorff metric is given by –

$d_H(A, B) = \max\left\{\sup_{a \in A} \inf_{b \in B} \| a - b \|, \sup_{\mathrm{b} \in \mathrm{B}} \inf_{a \in A} \| a - b \|\right\}$, $\left(P_k(R^m), d_H\right)$ is a complete and separable metric space[8].

Let $\mathrm{S_F}$ be the set of all selections of $F(.)$ that belong to the space of summable functions –

$$S_{R^m}(J) = \left\{ f \in R^m / \sum_J \| f \| < \infty \right\}, \text{ i.e.,}$$
$$\mathrm{S_F} = \left\{ f(.) \in S_{R^m}(J) \ / \ f(t) \in F(t) \ a.e. \right\}$$

We define the summable function for multi-valued functions as follows –

$$\sum_J F(n) = \left\{ \sum_J f(n), f(.) \in S_F \right\}$$

Define $E^m = \{u : \boldsymbol{R^m} \to [0,\ 1]\}$, u is normal, fuzzy convex, upper semi continuous and support $[u]^0$ is compact.

For $0<\alpha\le 1$, denote $[u]^{\alpha}=\{x\in R^m / u(x)\ge\alpha\}$, then the α-level set $[u]^{\alpha}\in P_k(R^m)$ for all $0<\alpha\le 1$. Define $D: E^m\times E^m\to[0,\infty)$ by the equation $D(u,v)=\sup_{0\le\alpha\le 1} d_H\left([u]^{\alpha},[v]^{\alpha}\right)$, where d_H is the Hausdorff metric defined in $P_k(R^m)$. The addition and scalar multiplication for any two fuzzy sets $u,v\in E^m$, and for any scalar $\lambda\in\mathbf{R}$

$[u+v]^{\alpha}=[u]^{\alpha}+[v]^{\alpha}, [\lambda \mathrm{u}]^{\alpha}=\lambda[u]^{\alpha}$, for $0\le\alpha\le 1$. Then (E^m,D) is a complete metric space. For properties on D and for results on fuzzy sets we refer to[9].

Definition 2.1. A fuzzy set valued function $F: J\to E^m$ is called summably bounded, if it is measurable and there exists a summable function h such that $\|y\|\le h(n)$ for all $y\in F_0(n)$.

A fuzzy set valued function $F: J\to E^m$ is called summably bounded if F_{α} is summable bounded for all α in [0, 1].

Theorem 2.1. If the fuzzy set valued mapping $F: J\to E^m$ is summably bounded, then $\sum_J F\in E^m$.

Proof: The proof is similar to proof of Theorem 3.1[8].

Definition 2.2. Let $F: J\to E^m$ be a fuzzy summably bounded mapping. The fuzzy summation of F over J denoted by $\sum_J F(n)$, is defined level wise by –

$$\left[\sum_J F(n)\right]^{\alpha}=\sum_J F_{\alpha}(n).$$

Theorem 2.2. If $F: J\to E^m$ is strongly measurable and summably bounded, then F is summable.

Proof: The proof is similar to the proof in[8].

Theorem 2.3. Let $F: J\to E^m$ be two fuzzy set valued functions which are fuzzy summable and $\lambda\in R.$. Then

(i) $\sum(F+G)=\sum F+\sum G$

(ii) $\sum\lambda F=\lambda\sum F$

(iii) $D(F,G)$ is summable

(iv) $D\left(\sum F,\sum G\right)=\sum D(F,G)$

Proof: (i) Let $\alpha\in[0,\ 1]$. Since F_α and G_α are compact convex valued, it follows that summation $\sum F_\alpha$ and $\sum G_\alpha$ are in S_F. Hence

$$\sum(F+G)_\alpha=\sum F_\alpha+\sum G_\alpha=\sum(F_\alpha+G_\alpha)$$

(ii) Let $\lambda\in R$ and $\alpha\in[0,1]$. Since F is compact convex closed, it follows that $\sum(\lambda F_\alpha)$ is in S_F. Hence

$$\sum(\lambda F)_\alpha=\sum\lambda F_\alpha=\lambda\sum F_\alpha$$

(iii) Let $\left\{f_n^\alpha/n=1,2,3,....\right\},\left\{g_n^\alpha/n=1,2,3,.....\right\}$ be a casting representation for F_α and G_α. By the definition of D

$D(F(n),G(n))=\sup\limits_{k\geq1}d_H\left(F_{\alpha_k}(n),G_{\alpha_k}(n)\right)$, where $\{\alpha_k/k=1,2,3,....\}$ is dense in [0, 1], is measurable. Further,

$D\left(F(n),G(n)\right)\leq D\left(F(n),\bar{0}\right)+D(G(n),\bar{0})\leq h_1(n)+h_2(n)$ where h_1 and h_2 are summable bounds for F and G respectively. Hence from Theorem 2.2, $D(F,G)$ is summable.

(iv) From the def of d_H, we have $d_H\left(\sum F_\alpha,\sum G_\alpha\right)\leq\sum d_H(F_\alpha,G_\alpha)$, and consequently

$$D\left(\sum F,\sum G\right)\leq\sup_{\alpha\in[0,1]}\sum d_H\left(F_\alpha,G_\alpha\right)\leq\sum\sup_{\alpha\in[0,1]}d_H(F_\alpha,G_\alpha)=\sum D(F,G).$$

Let $F:J\times E^m\to E^m$. Consider the fuzzy difference equation –

$$\Delta x(n) = f(n, x(n)),\ \mathrm{x}(0) = \mathrm{x}_0 \qquad \ldots(2.1)$$

Definition 2.3: A mapping $x : J \to E^m$ is a fuzzy solution to (2.1), if it satisfies the equation

$$x(n) = x_0 + \sum_{k=0}^{k=n-1} F(k, x(k)),\ \text{for all } n \in J$$

The fundamental matrix of the equation

$$x(n+1) = A(n)x(n), \quad \text{for } n \geq 0 \qquad \ldots(2.2)$$

is defined by $Y(n) = \begin{cases} A(n-1)(n-2)(n-3)......A(1)A(0);\ \mathrm{n} > 0 \\ I_m;\ \mathrm{n} = 0 \end{cases}$ and the solution (2.2) with $x(0) = x_0$ is $x(n) = Y(n)x_0$.

Formation of fuzzy difference control system

In this section we show that the deterministic control system (1.1) and (1.2) with fuzzy inputs $u(n)$ determines a fuzzy difference control system.

For $0 < \alpha \leq 1$, let $[u(n)]^{\alpha}$ be the α-level set of $u(n)$. Consider difference inclusions

$$x_{\alpha}(n+1) \in A(n)x_{\alpha}(n) + B(n)[u(n)]^{\alpha},\ \mathrm{x}(0) = 0,\ \mathrm{n} \in \mathrm{J} \qquad \ldots(3.1)$$

Let x^{α} be the solution set of inclusion (3.1).

Claim (i) $[x(n)]^{\alpha} \in P_k(\mathbf{R}^{\mathrm{m}})$ for all $n \in J$. First we prove that x^{α} is non-empty, compact and convex in the set of all functions from J to $\mathbf{R}^{\mathrm{m}}$, Since $[u(n)]^{\alpha}$ has measurable selection, we have x^{α} non-empty. Let $K_1 = \max_{n \in J} \| Y(n) \|,\ \mathrm{K}_2 = \max_{n \in J} \| Y^{-1}(n) \|,$

$$M = \max_{n \in J} \left\{ \| u(n) \|, u(n) \in | u(n) |^{\alpha} \right\},\ \mathrm{N} = \max_{n \in J} \| B(n) \|,$$

For any $x \in X^{\alpha}$, there is a selection $u(n) \in [u(n)]^{\alpha}$ such that

$$x(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u(k)$$

It implies that

$$\| x(n) \| \leq \| Y(n)x_0 \| + \| \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u(k) \| \leq K_1 \| x_0 \| + LK_1K_2MN$$

Thus x^{α} is bounded. To prove x^{α} is compact, it is sufficient to prove that it is closed. Let $\{x_m\} \in x^{\alpha}$ and $\{x_m\} \to x.$. For each $\{x_m\} \in x^{\alpha}$, there is a $u_m \in [u(n)]^{\alpha}$ such that

$$x_m(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u_m(k)$$

Since $u_m \in [u(n)]^{\alpha}$ is closed, then there exists a sub sequence $\{u_{m_j}\}$ of $\{\mathrm{u_m}\}$ converging weakly to $u \in [u(n)]^{\alpha}$ '. From Mazur's theorem, there exists a sequence of numbers $\lambda_j \succ 0$, $\sum \lambda_{\mathrm{j}} = 1$ such that $\sum \lambda_j u_{mj}$ converges strongly to u. Thus we have

$$\sum \lambda_j x_{mj}(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k) \sum \lambda_j u_{mj}(k) \qquad \ldots(3.2)$$

Taking the limit as $j \to \infty$, on both sides of (3.2), we have

$$x(n) = Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)u(k)$$

Thus $x(n) \in x^{\alpha}$, for all $n \in J$. Hence x^{α} is closed. Now we prove x^{α} is convex.

Let $x_1, x_2 \in x^{\alpha}$, then there exists $u_1, u_2 \in [u(n)]^{\alpha}$ such that

$$x_1(n+1) = A(n)x_1(n) + B(n)u_1(n),\ x_2(n+1) = A(n)x_2(n) + B(n)u_2(n),$$

Let $x = \lambda x_1 + (1-\lambda)x_2,\ \ 0 \leq \lambda \leq 1$, then

$$\begin{aligned} x(n+1) &= \lambda x_1(n+1) + (1-\lambda)x_2(n+1) \\ &= \lambda\big(A(n)x_1(n) + B(n)u_1(n)\big) + (1-\lambda)\big(A(n)x_2(n) + B(n)u_2(n)\big) \\ &= A(n)(\lambda x_1(n) + (1-\lambda)x_2(n)) + B(n)(\lambda u_1(n) + (1-\lambda)u_2(n)). \end{aligned}$$

Since $[u(n)]^{\alpha}$ is convex, $\lambda u_1(n) + (1-\lambda)u_2(n) \in [u(n)]^{\alpha}$, we have

$$x(n+1) = A(n)x(n) + B(n)[u(n)]^{\alpha}$$

Hence $x \in x^{\alpha}$. Therefore x^{α} is convex. Thus $[x(n)]^{\alpha} \in P_k(R^n)$, for every $n \in J$.

Claim (ii) $[x(n)]^{\alpha_2} \subset [x(n)]^{\alpha_1}$, for all $0 \le \alpha_1 \le \alpha_2$. Let $0 \le \alpha_1 \le \alpha_2$. Since u is a fuzzy set, $[u(n)]^{\alpha_2} \subset [u(n)]^{\alpha_1}$, *then* $S_{[u(n)]^{\alpha_2}} \subset S_{[u(n)]^{\alpha_1}}$ and the following inclusion

$$x_{\alpha_2}(n+1) \in A(n)x_{\alpha_2}(n) + B(n)[u(n)]^{\alpha_2} \subset A(n)x_{\alpha_1}(n) + B(n)[u(n)]^{\alpha_1}$$

It implies that

$$x_{\alpha_2}(n) \in Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]^{\alpha_2}}$$
$$\subset Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]^{\alpha_1}}$$

Thus $[x]^{\alpha_2} \subset [x]^{\alpha_1}$, and hence $[x(n)]^{\alpha_2} \subset [x(n)]^{\alpha_1}$, for all $n \in J$.

Claim (iii) If $\{\alpha_r\}$ is a non-decreasing sequence converging to $\alpha > 0$, then $x^{\alpha}(n) = \bigcap_{r \ge 1} x^{\alpha_r}(n)$.

Since u is a fuzzy set, we have $[u(n)]^{\alpha} = \bigcap_{r \ge 1} [u(n)]^{\alpha_r}$, which implies that $S_{[u(n)]^{\alpha}} = S_{\bigcap_{r \ge 1} [u(n)]^{\alpha_r}}$.

Thus $x_{\alpha}(n+1) \in A(n)x_{\alpha}(n) + B(n)[u(n)]^{\alpha} = A(n)x_{\alpha}(n) + B(n)\bigcap_{r \ge 1} |u(n)|^{\alpha_r}$

$$\subset A(n)x_{\alpha}(n) + B(n)[u(n)]^{\alpha_r}, \quad r = 1, 2, 3, \ldots$$

Thus, $x_{\alpha} \subset x_{\alpha_n}$, $r = 1, 2, 3, \ldots$ which implies that $x_{\alpha} \subset \bigcap_{r \ge 1} x_{\alpha_r}$.

Let x be the solution of the inclusions

$$x_{\alpha_k}(n+1) \in A(n)x_{\alpha_r}(n) + B(n)[u(n)]^{\alpha_r}, \; \mathrm{r} \geq 1$$

Then $x(n) \in Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]}{}^{\alpha_r}$, it follows that

$$x(n) \in Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k) \bigcap_{r\geq 1} S_{[u(n)]}{}^{\alpha_r}$$

$$= Y(n)x_0 + \sum_{k=0}^{n-1} Y(n)Y^{-1}(k+1)B(k)S_{[u(n)]}{}^{\alpha_r}$$

This implies that $x \in x^{\alpha}$.

Therefore $\bigcap_{r\geq 1} x^{\alpha_r} \subset x^{\alpha}. From$ (3) we have $\mathrm{x}^{\alpha} = \bigcap_{r\geq 1} x^{\alpha_r}$, and hence $\mathrm{x}^{\alpha}(n) = \bigcap_{r\geq 1} x^{\alpha_r}(n)$ for all $n \in J$.

Hence there exists $x(n) \in E^m$ on J such that $x^{\alpha}(n)$ is a solution set to the difference inclusions (3.1). Hence the solution (1.1) and (1.2) is a fuzzy difference control system.

Observability of fuzzy difference control systems

Now, we discuss the concept of likely observability and obtain sufficient condition for the observability of fuzzy difference control system (1.1), (1.2).

Definition 4.1. The state $x_0 \neq 0$ of a fuzzy system (1.1), (1.2) is said to be likely observable at level α on J if the knowledge of the α-level input $u(n)$ and the α-level output $v(n)$ over J sufficient to determine the range of the initial state x_0.

We will present the sufficient condition for the system (1.1), (1.2) to be observable. The norm of the matrix $A(n) = [a_{ij}(n)]$ defined by $\| A(n) \| = \max_{i,j} | a_{ij}(n) |$ and maximum norm $\| A \| = \max_{n\in J} \| A(n) \|$.

Theorem 4.1. The fuzzy system (1.1), (1.2) is likely observable at level α on J if $C(L)Y(L)$ is non-singular. Further, let $u_0(n)$ and $y_0(n)$ be the centre points of $u(n)$ and $y(n)$ respectively, and let x^{α} be the possible initial point on α-level, then the range estimate for the initial value α-level is given by (4.1).

$$||x_\alpha - x_0|| \le ||[C(L)Y(L)]^{-1}||\{\max \ ||\overline{y}_\alpha(L) - y_0(L)|| + ||D(L)|| \max_{\overline{u}_\alpha(L)\in u_\alpha(L)} ||\overline{u}_\alpha(L) - u_0(L)|| + ||C(L)||\,||Y||\,||Y^{-1}||\,||B|| \sum_{k=0}^{n-1} \max_{\overline{u}_\alpha(L)\in u_\alpha(L)} ||\overline{u}_\alpha(L) - u_0(L)||\}.$$

Proof: Let x_α be the solution set of (1.1), (1.2) we have

$$x_\alpha(L) = Y(L)x_0 + \sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_\alpha(k), \text{ then}$$

$$y_\alpha(L) = C(L)\left[Y(L)x_0 + \sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_\alpha(k)\right] + D(L)u_\alpha(L),$$

where $y_\alpha(L)$ is the α-level set of $y(L)$. Thus,

$$C(L)Y(L)x_0 \in y_\alpha(L) - D(L)u_\alpha(L) - C(L)\sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_\alpha(k).$$

Let x_α be the possible initial value, then we can write the above equation as

$$C(L)Y(L)x_\alpha \in y_\alpha(L) - D(L)u_\alpha(L) - C(L)\sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_\alpha(k). \qquad \ldots(4.2)$$

Since u_0, y_0 are the centre points of u, y respectively, we have

$$|C(L)Y(L)|\,x_0 = y_0 - D(L)u_0 - C(L)\sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_0. \qquad \ldots(4.3)$$

From (4.2), (4.3), and the fact that $C(L)Y(L)$ is non-singular, we can estimate the distance between x_α and x_0 as follows:

$$||C(L)Y(L)(x_\alpha - x_0)|| \le \max d_H\Big(y_0(L) - D(L)u_0(L) - C(L)\sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_0(k), \; Y_\alpha(L) - D(L)u_\alpha(L) - C(L)\sum_{k=0}^{L-1} Y(L)Y^{-1}(k+1)B(k)u_\alpha(k)\Big)$$

$$\leq \max_{\bar{y}_\alpha(L)\in y_\alpha(L)} ||\bar{y}_\alpha(L) - y_0(L)|| + ||D(L)|| \max_{\bar{u}_\alpha(L)\in u_\alpha(L)} ||\bar{u}_\alpha(L) - u_0(L)||$$
$$+ ||C(L)|| \max_{n\in J} ||Y(n)|| \max_{n\in J} ||Y^{-1}(n)|| \max_{n\in J} ||B(n)||$$
$$\sum_{k=0}^{L-1} \max_{\bar{u}_\alpha(L)\in u_\alpha(L)} ||\bar{u}_\alpha(L) - u_0(L)||.$$

Example 4.1. Consider the fuzzy difference control system (1.1), (1.2) with

$$A(n) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},\ B(n) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C(n) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ D(n) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$u_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \mathrm{y}_0 = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix},\ \mathrm{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Also assume that the α-level sets of input u (n) and output y(n) are

$$u_\alpha = \begin{bmatrix} [\alpha - 1, 1 - \alpha] \\ (0.1)(\alpha - 1),\ (0.1)(1 - \alpha) \end{bmatrix},\ \mathrm{y}_\alpha = \begin{bmatrix} \left[-\sqrt{1-\alpha}, \sqrt{1-\alpha}\right] \\ \left[\frac{1}{2} - \sqrt{1-\alpha},\ \frac{1}{2} + \sqrt{1-\alpha}\right] \end{bmatrix}$$

Then the fundamental matrix of (2.2) is $Y(n) = \begin{bmatrix} \cos\left(\frac{n\pi}{2}\right) & -\sin\left(\frac{n\pi}{2}\right) \\ \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \end{bmatrix}$

Clearly, the centre points of $u(n)$ and $y(n)$ are $u_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \mathrm{y}_0 = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$

And also

$\| \mathrm{Y} \| = \| \mathrm{Y}^{-1} \| = 1,\ \| [\mathrm{C(L)Y(L)}]^{-1} \| = 1,\ \| \mathrm{u}_\alpha - \mathrm{u}_0 \| = 1 - \alpha,$
$\| \mathrm{y}_\alpha - \mathrm{y}_0 \| = \sqrt{1-\alpha},\ \| \mathrm{C} \| = 1,\ \| \mathrm{D} \| = 0.$

Substituting these values in (4.1), we have

$$\| x_\alpha - x_0 \| \leq \sqrt{1-\alpha} + L(1-\alpha)$$

From the above inequality, it is easily observed that as L becomes larger, our estimated area becomes larger. This means that as L increases it is difficult to determine the initial value. As $\alpha \to 1$, then $\| x_\alpha - x_0 \| \to 0$, i.e., x_α approaches the initial value x_0. The above example illustrates the significance of our method by which we can determine the range of initial value without solving the FDCS.

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