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DOUBLE DIFFUSIVE MIXED CONVECTION AT A VERTICAL PLATE IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT

An isothermal permeable plate is assumed to be immersed vertically in a homogeneous viscous electrically conducting fluid containing a concentration species. The fluid is assumed to be moving with a constant velocity parallel to the plate. Assuming fluid viscosity to be a function of temperature and taking into consideration the effect of Ohmic heating, double diffusive magneto hydrodynamic (MHD) mixed convection flow and heat transfer at the vertical plate is studied numerically. Assuming the flow to be two-dimensional and introducing a similarity variable, the governing equations of the problem are reduced to a set of non-linear ordinary differential equations. The equations subject to appropriate boundary conditions are solved by Nachtsheim-Swigert scheme together with a Shooting method. Both the cases of assisting and opposing buoyancies are considered. The effects of magnetic field, Ohmic heating and mass diffusion on different flow and heat transfer characteristics like skin friction, Nusselt number and Sherwood number are presented graphically and discussed. Skin friction, and Nusselt number are found to diminish and Sherwood number is found to increase with increasing values of the Schmidt number. In the assisting flow case the Skin friction, the Nusselt number as well as the Sherwood number increase while in opposing flow case, they diminish with different parameters.

Key words: Double diffusive mixed convection, MHD, Ohmic heating.

INTRODUCTION

Rossow[1] made a detailed analysis of the flow of electrically conducting fluids over a flat plate in the presence of a transverse magnetic field. Due to the magnetic field, the conducting fluid experiences a force that resists its motion across the magnetic lines of force. This causes stabilization of the flow. In fact, the stabilizing force is due to an induced

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electric current. Hartmann and Lazarus[2] pointed out that a turbulent stream could be stabilized so that it returns to laminar flow by the effect of an applied magnetic field.

Heat and mass transfer studies have applications in many fields of science and technology including chemical engineering. There is a plethora of papers in literature on different aspects of convective heat and mass transfer. Some of the papers discussed free convection while others discussed mixed convection. In this article, only a few relevant articles on mixed convection are quoted.

Mahmoud[3] studied the effects of variable viscosity and chemical reaction on mixed convection heat and mass transfer at a semi infinite vertical plate. Taking the plate to be porous, assuming a similarity transformation, the governing equations of the problem were reduced to a set of non linear ordinary differential equations and solved numerically by the use of a shooting method. The variations in the flow and heat transfer characteristics including local skin friction coefficient, the local Nusselt number and the local Sherwood number with the different non-dimensional parameters were presented and discussed.

Patil et al.[4] made an extensive study of steady double diffusive mixed convection flow over a continuously moving vertical plate in the presence of internal heat generation and chemical reaction. Using suitable transformations, the governing equations of flow, heat and mass transfer were transformed into non-similar form and solved by the use of an implicit finite difference scheme together with quasi linearization. Computations were made for a wide range of values of the parameters of the problem including the buoyancy ratio parameter, Schmidt number, Prandtl number and chemical reaction parameter. Important results pertaining to the flow, heat and mass transfer characteristics for the cases of assisting buoyancies as well as opposing buoyancies were presented and discussed as functions of parameters of the problem.

Geetha and Moorthy[5] studied the effects of variable viscosity, chemical reaction and thermal stratification on mixed-convection heat and mass transfer at a semi-infinite vertical plate. Using a similarity transformation, the governing equations of the problem were transformed into ordinary differential equations and were solved by Runge Kutta-Gill method together with a shooting technique. The effects of different parameters of the problem on the flow and heat transfer characteristics were presented and discussed.

In the present analysis, the effects of chemical reaction, magnetic field and Ohmic heating on double diffusive MHD mixed convection at a vertical plate are studied numerically. The emphasis is mainly on the effects of magnetic field, mass diffusion and chemical reaction. Under certain assumptions and the use of a similarity variable the governing equations are transformed into coupled non-linear ordinary differential equations and solved by the use of Nachtsheim–Swigert scheme together with a shooting technique. Both the cases of assisting and opposing buoyancies are discussed. The flow, heat and mass transfer characteristics are determined as functions of different non-dimensional parameters of the problem. It is well known that pure water is not a conductor of electric current, but slightly saline water or water with impurities is a conductor of electric current. Liquid metals are also known to be excellent conductors of electric current. In this paper, numerical computations for two values of the Prandtl number namely Pr = 7.0 (water) and Pr = 0.064 (Lithium) are presented. Certain qualitatively and quantitatively important results are presented and discussed.

Formulation and solution

Let a vertical permeable plate be immersed vertically in a homogeneous electrically conducting viscous fluid containing a concentration species. Let the fluid flow with a constant velocity (u_∞) parallel to the plate. Let x-axis be taken vertically upwards along the plate and y-axis perpendicular to it. Let a magnetic field of intensity B_o be applied transverse to the plate, and let viscosity of the fluid vary as an inverse linear function of temperature as –

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left\{1 + \alpha\,(T - T_\infty)\right\} \qquad ...(1)$$

Using Boussinesque's approximation, taking u, ν to be the velocity components of the fluid along x-axis and y-axis, T to be temperature of the fluid and C to be concentration of the species, the equations governing the boundary layer MHD Mixed convection flow, heat and mass transfer are written as –

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots(2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta\,(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho_\infty}u \qquad ...(3)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\,\frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2 \qquad ...(4)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\,\frac{\partial^2 C}{\partial y^2} - K_1\,(C - C_\infty) \qquad \dots(5)$$

The relevant boundary conditions are –

$$\text{at } y = 0,\ u = 0,\ v = -V_w,\ C = C_w,\ T = T_w$$

$$\text{as } y \to \infty,\ u \to u_\infty,\ C \to C_\infty,\ T \to T_\infty \qquad \ldots(6)$$

Let stream function ψ be introduced through the relations $u = \frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$ so that the equation of continuity is identically satisfied. A similarity variable η and non dimensional functions f, θ, ϕ are introduced into the governing equations through the relations.

$$\eta = \sqrt{\frac{u_\infty}{\nu_\infty}}, \frac{y}{\sqrt{x}}$$

$$\psi = \sqrt{v_\infty, u_\infty, x}.f(\eta)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}$$

Then the governing equations (2)-(3) get transformed into the non linear ordinary differential equations.

$$(\theta - \theta_r) f''' - f'' \theta' - \frac{(\theta - \theta_r)^2}{2\theta_r} ff'' - \lambda \frac{(\theta - \theta_r)^2}{\theta_r} (\theta + N\text{Ø}) + \frac{(\theta - \theta_r)^2}{\theta_r} M f' = 0 \qquad \ldots(7)$$

$$\theta'' + \frac{1}{2} \Pr f \theta' + M P_r E_c f'^2 = 0 \qquad \ldots(8)$$

$$\text{Ø}'' + \frac{1}{2} Sc f \text{Ø}' - \gamma Sc \text{Ø} = 0 \qquad \ldots(9)$$

The relevant boundary conditions in terms of f, θ, Ø are –

$$\text{at } \eta = 0,\ f = v_w,\ f' = 0,\ \theta = 1,\ \text{Ø} = 1$$

$$\text{as } \eta \to \infty,\ f' \to 1,\ \theta \to 0,\ \text{Ø} \to 0 \qquad \ldots(10)$$

In these equations and boundary conditions,

$\theta_r = \dfrac{-1}{\alpha\,(T_w - T_\infty)}$ is the viscosity variation parameter,

$v_w = \dfrac{2\sqrt{x}\,V_w}{\sqrt{v_\infty\, u_\infty}}$ is the suction/injection parameter,

$Gr_x = \dfrac{g\,\beta\,(T_w - T_\infty)\,x^3}{v_\infty^2}$ is the local Grashof number,

$Grc_x = \dfrac{g\,\beta^*(C_w - C_\infty)\,x^3}{v_\infty^2}$ is the Grashof number for the concentration species,

$M_x = \sqrt{\dfrac{\sigma\,B_o^2 x^2}{\mu_\infty}}$ is the Magnetic Reynolds number,

$Pr = \dfrac{\mu_\infty cp}{k}$ is the Prandtl number,

$Re_x = \dfrac{u_\infty x}{v_\infty}$ is the local Reynolds number,

$Ec = \dfrac{u_\infty^2}{cp\,(T_w - T_\infty)}$ is the Eckert number,

$\gamma = \dfrac{K_1 x}{u_\infty}$ is the modified chemical reaction parameter,

$Sc = \dfrac{\nu_\infty}{D}$ is the Schmidt number,

$N = \dfrac{\beta^*(C_w - C_\infty)}{\beta\,(T_w - T_\infty)}$ is the buoyancy Ratio parameter,

$M = \dfrac{M_x^2}{Re_x}$ is the magnetic parameter and

$\lambda = \dfrac{Gr_x}{Re_x^2}$ is the parameter that describes relative importance of natural convection over forced convection and the other symbols have their usual meanings .

Solution procedure

Nachtsheim-swigert scheme is an efficient numerical technique used to solve boundary layer type asymptotic problems. The scheme, together with a shooting technique is utilized in the present analysis to obtain the numerical solutions of equations (7), (8) and (9) subject to the boundary conditions (10) for different values of parameters of the problem.

RESULTS AND DISCUSSION

The physical quantities of interest in applications are the Drag coefficient, the Nusselt number and the Sherwood number. It is a well known fact that the Drag coefficient at the plate is directly proportional to the skin friction, $f''(0)$; the Nusselt number is proportional to the wall transfer rate, $'-\theta'(0)'$; and the Sherwood number is proportional to the mass transfer rate at the wall, $'-\O'(0)'$. For sake of simplicity in our discussion, we refer to $'-\theta'(0)'$ as the Nusselt number and $'-\O'(0)'$ as the Sherwood number.

It may be noted that the flow and heat and mass transfer characteristics depend upon a number of non-dimensional parameters, N- the ratio of buoyancy forces, Pr-the Prandtl number, θ_r- the viscosity variation parameter, M- the magnetic parameter, γ- the chemical reaction parameter, ν_w- the suction/injection parameter, Sc-the Schmidt number, Ec-the parameter describing the Ohming heating effect and λ-the ratio of the free convective to the square of the forced convective forces.

In view of the available literature and in view of the large number of non-dimensional parameters, the present work is restricted to some specific numerical values of ν_w, θ_r and a range of values of the other parameters.

Although it is a mathematical analysis, to make at a bit closure to practical situations, since the fluids are expected to be good conductors of the electricity in MHD flows, in the numerical computations, the Prandtl number Pr is assigned two values, 7.0 and 0.064. These values of Pr correspond to water and lithium, respectively. Though pure water is not at all a conductor of electricity, water with slight impurities is a conductor and lithium being a liquid metal is also a good conductor of electricity. The Schmidt number Sc is given a range of values, which correspond to a variety of concentration species. The buoyancy ratio N is assigned both positive and negative values, which correspond to the cases of assisting buoyancies and opposing buoyancies. The other parameter λ is assigned a few numerical values indicating relatively larger or smaller effect of free convective forces over forced convective forces.

It may be noted that computations are made for a large value of θ_r, i.e., 500 since viscosity variations for the fluids under consideration, with temperature is negligible. The discussion mainly is divided into two parts – Case (i) when Pr = 7.0 and Case (ii) when Pr = 0.064

Case (i) Pr = 7.0 (Water)

The plots of the skin friction versus the buoyancy ratio parameter N are presented in Fig. 1. For all values of the parameters, the skin friction is found to increase with increasing values of the parameter N. For negative values of N, that is in the case of opposing buoyancies, as can be expected, the skin friction assumes smaller numerical values as compared to the case of assisting buoyancies, i.e., when N takes positive values. Further skin friction takes increasing values with increasing values of λ and this is also an expected result, since skin friction can increase with increasing buoyancy forces.

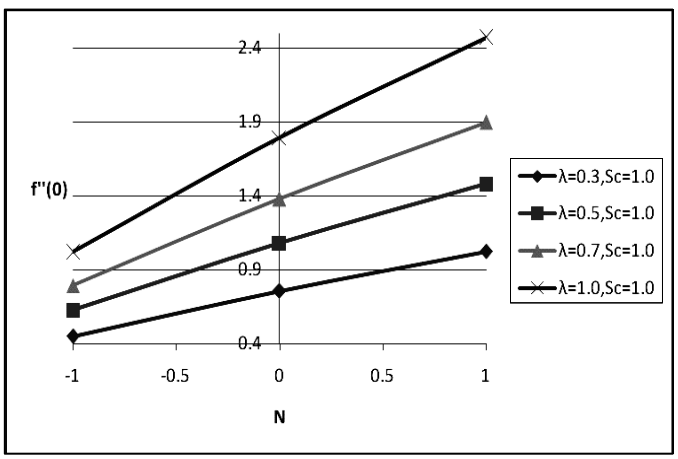

Fig. 1: Variations in f′′ (0) with the buoyancy ratio parameter N for Pr = 7.0

The variations in the wall heat transfer rate ′− θ′ (0)′ or the Nusselt number and the variations in the Sherwood number ′− Ø′ (0)′, with different parameters are presented in Figs. 2 and 3, respectively. Both the Nusselt number and the Sherwood number, like skin friction, increase with increasing values of the parameter, N though, the rate of increase is not as high as in the case of skin friction. The Nusselt number and the Sherwood number increase with increasing values of λ also.

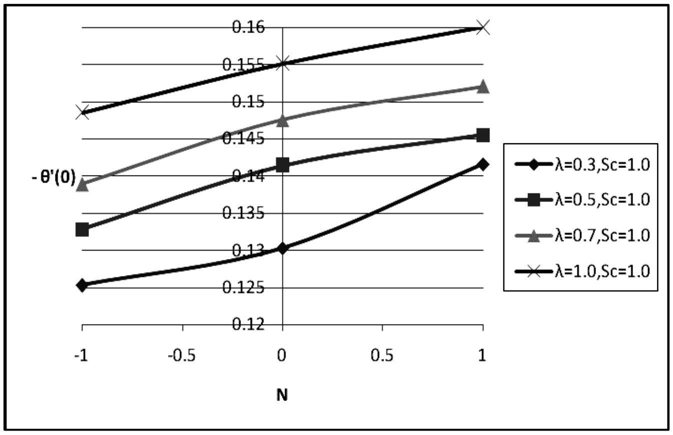

Fig. 2: Variations in ′− θ′ (0)′ with the buoyancy ratio parameter N for Pr = 7.0

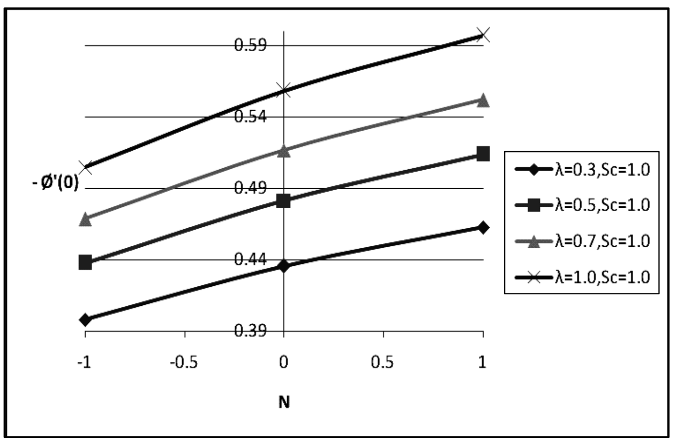

Fig. 3: Variations in ′− Ø′ (0)′ with the buoyancy ratio parameter N for Pr = 7.0

Variations in skin friction (f′′ (0)), the Nusselt number (′− θ′ (0)′) and the Sherwood number (′− Ø′ (0)′) with Schmidt number (Sc) are presented in Figs. 4, 5 and 6, respectively. The skin friction can be seen to diminish with increasing values of Sc and approach a constant value for large values of Sc. The diminishing nature is very sharp up to ‘Sc = 10’ whereas it is not so sharp beyond ‘Sc = 10’. Further skin friction assumes smaller values when M and EC assumes larger values or when the intensity of the magnetic field and the associated Ohmic heating effect increase and also when the chemical reaction parameter (γ) increases.

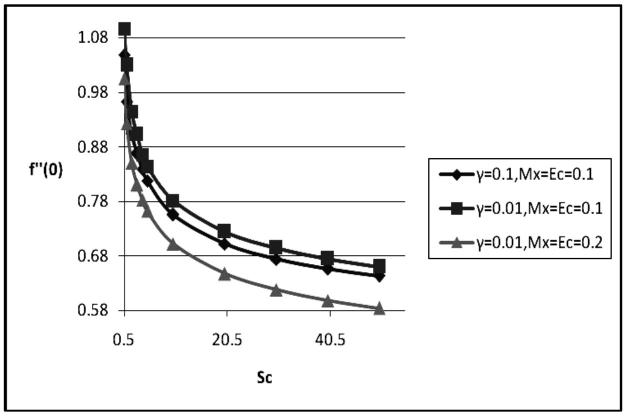

Fig. 4: Plots of f′′ (0) versus the Schmidt number Sc for Pr = 7.0

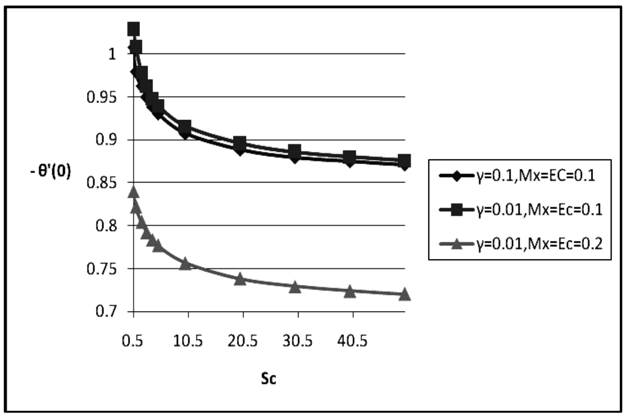

Fig. 5: Plots of ′− θ′ (0)′ versus the Schmidt number Sc for Pr = 7.0

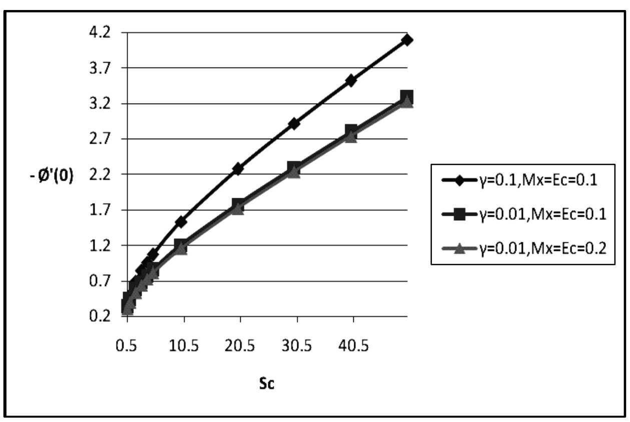

Fig. 6: Plots of ′− Ø′ (0)′ versus the Schmidt number Sc for Pr = 7.0

Plots of the Nusselt number are qualitatively similar to those of skin friction indicating that variations in Nusselt number with Sc, γ, M and Ec are similar to those of the skin friction (Figs. 4, 5). However, Sherwood number can be seen to increase with increasing values of the Schmidt number (Sc) and increases almost linearly with Sc (Fig. 6). Sherwood number can be seen to increase with increasing values of the chemical reaction parameter (γ) and decrease with increasing intensity of the magnetic field and Ohmic heating.

Case (ii) Pr = 0.064 (Lithium)

Variations in the skin friction ($f''(0)$), the Nusselt number ($'-\theta'(0)'$) and the Sherwood number ($-\O'(0)$) are presented in Figs. 7, 8 and 9, respectively.

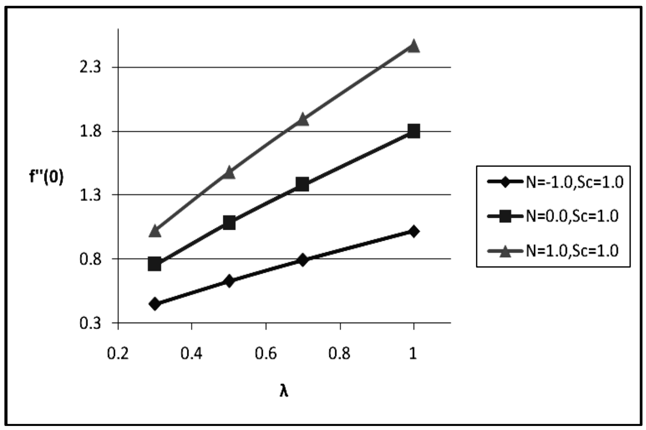

Fig. 7: Plots of $f''(0)$ versus the parameter λ for Pr = 0.064

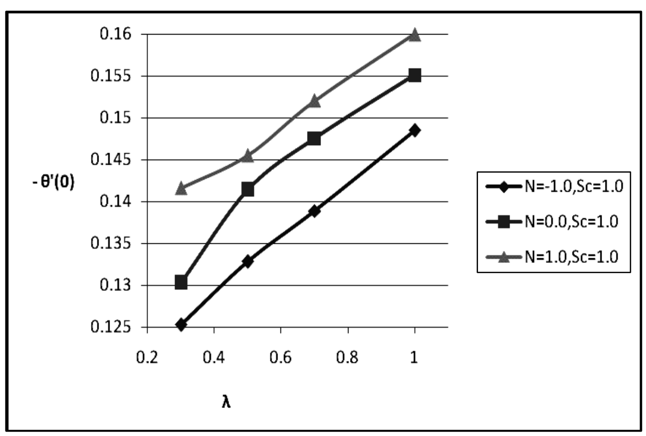

Fig. 8: Plots of $'-\theta'(0)'$ versus the parameter λ for Pr = 0.064

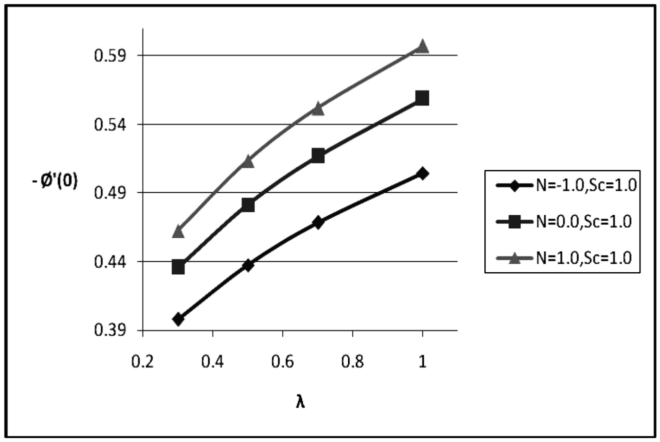

Fig. 9: Plots of ′− Ø′ (0)′ versus the parameter λ for Pr = 0.064

From Fig. 7, it may be noted that the skin friction increases with increasing values of 'λ', as the relative effect of free convection over forced convection increases. It may be also noted that skin friction assumes increasing values with increasing values of N. For negative values of N, that is in the case of opposing buoyancies, skin friction assumes smaller numerical values and for positive values of N, that is in the case of assisting buoyancies, skin friction assumes relatively larger numerical values. From Figs. 8 and 9, one may notice that the behaviors of the Nusselt number and the Sherwood number with the parameters are similar to those of the skin friction.

Variations in skin friction (f′′ (0)), the Nusselt number (− θ′ (0)) and the Sherwood number (− Ø′ (0)) with different parameters are presented in Figs. 10, 11 and 12, respectively.

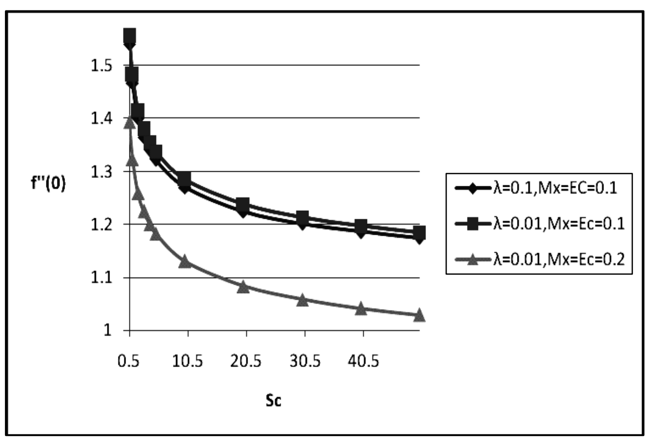

Fig. 10: Plots of f′′ (0) versus the Schmidt number Sc for Pr = 0.064

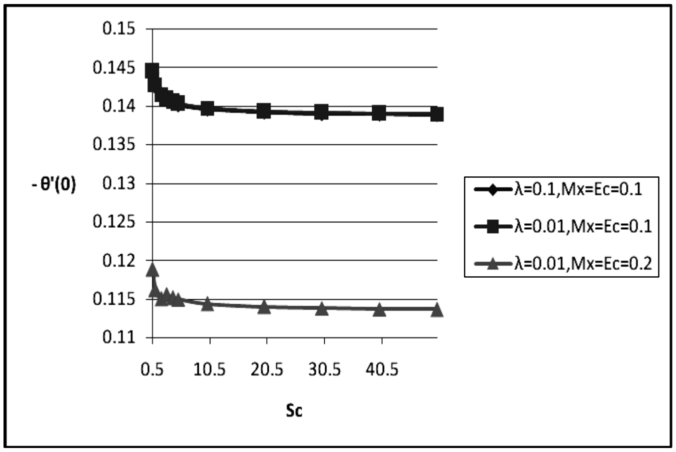

Fig. 11: Plots of $'-\theta'(0)'$ versus the Schmidt number Sc for Pr = 0.064

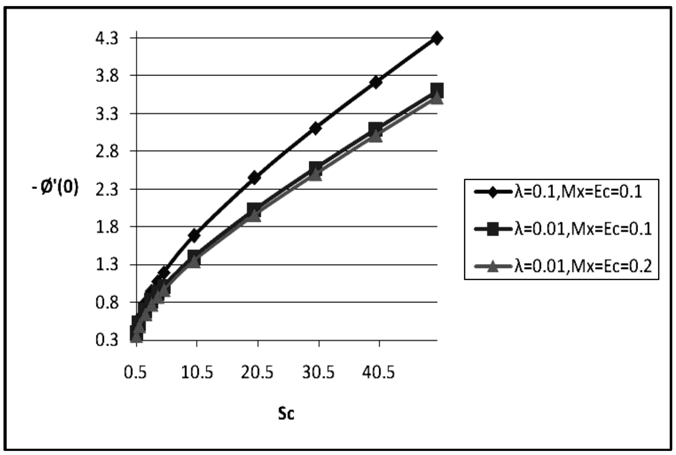

Fig. 12: Plots of $'-\varnothing'(0)'$ versus the Schmidt number Sc for Pr = 0.064

On a comparison of Figs. 10 to 12 for lithium (Pr = 0.064) with the corresponding Figs. 4 to 6 for water (Pr = 7.0), one may notice that the variations in flow heat and mass transfer characteristics for both the fluids are qualitatively similar. However, skin friction and the Nusselt number for lithium assume smaller numerical values than those for water.

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