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COMPARISON OF KALMAN AND EXTENDED KALMAN FILTER IN LEVEL-TEMPERATURE CASCADED PROCESS

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ABSTRACT

The aim of this work is to maintain the interacting liquid level and temperature parameter at a desired level. This paper presents decoupling, linearization algorithm (Hirschorn's algorithm) and Kalman filter (KF) and Extended Kalman filter (EKF) for an approximated model of an interacting thermal non-linear process. The nonlinear interacting system is converted into linear non interacting system using decoupling and linearization algorithm. KF and EKF are then designed to estimate the system parameters namely level and temperature for a non-interacting linear Multi Input Multi Output (MIMO) system. Performance of Extended Kalman filter was found to be better. The obtained estimated error for the plant using EKF is less when compared to KF.

Key words: Multivariable process control, Hirschorn's algorithms, Kalman filter, Extended Kalman filter.

INTRODUCTION

The chemical processes hold non-linear dynamic characteristics and the design of controller for a non-linear chemical process involves linearizing the process model around its steady state operating point and applying the linear control theory. However, decoupling and linearization control[1] theory is developed under the assumption that the process model is known exactly. Therefore, if there is a difference between the real process and the process model, application of this theory will give unsatisfactory results. In case, the degree of mismatch is there in the chemical process, it is sufficient to add external controllers to compensate the mismatch. The state feedback law is then applied to the nonlinear process. The resulting control structure is called the Globally Linearizing Control (GLC) structure. This methodology is tried here for a MIMO nonlinear system having equal number of inputs and outputs. For a linear non interacting MIMO system, KF and EKF are designed from control point of view. Recently there have been many researchers aiming to simultaneously

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estimate the system parameter and the unknown input. The estimation of parameter is important in many engineering applications.

Level-temperature process setup

Application to chemical process control - Level and Temperature Cascaded Process

The non-linear system is defined as

$$\dot{x} = f(x) + g(x)u \quad \ldots(1)$$

$$y = h(x) \quad \ldots(2)$$

Where 'x' is the state vector of dimension n,

'u' is an input vector of dimension m,

'y' is an output vector of dimension of p,

f(x) is a smooth function,

h(x) is a (p,1) vector with a row element $h_j(x)$ also a smooth function and g(x) is an (n, m) matrix with elements of each column being $g_j(x)$.

The model is of a liquid level and temperature process[2] is shown in Figure 1.

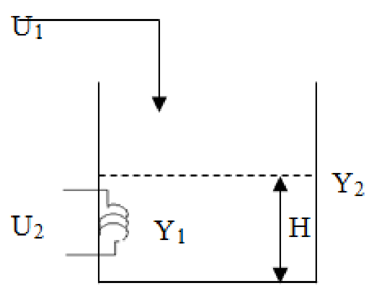

Fig. 1: Schematic diagram for the level and temperature control process

Here (a) U_1 is the feed flow rate of the liquid (b) U_2 is the heater input (c) Y_1 is the level sensor output (d) Y_2 is the temperature sensor output (e) H is height of the liquid.

The mathematical equations from the above system is –

$$\frac{dx_1}{dt} = -\left(\frac{k}{s}\right)x_1^{\frac{1}{2}} + \frac{1}{s}u_1 \qquad \ldots(3)$$

$$\frac{dx_2}{dt} = \left(\frac{T_o - x_2}{sx_1}\right)u_1 + \left(\frac{1}{c_p \varsigma s x_1}\right)u_2 \qquad \ldots(4)$$

$y_1 = x_1$

$y_2 = x_2$

where

x_1 and x_2 are the liquid level and the temperature in the tank, respectively. u_1 and u_2 are the feed flow rate to the tank and the heat flow rate from the heater, respectively. The feed flow rate and heat flow rate are constrained as $0 <= u_1 <= 22\ cm^3s^{-1}$ and $0 <= u_2 <= 2700\ Js^{-1}$.k = Constant Coefficient, 1.8. S = Cross sectional area, 191 cm^2.x_1 = Liquid level in cm. x_2 = Liquid temperature, °C. To = Temperature of the feed 18°C.Cp = Specific heat, = 4.2 $J^{-1}K^{-1}$.ς = density of the liquid. Simulations were carried out and closed loop response was obtained. The performance of the proposed algorithm was evaluated by extensive numerical simulations. A standard Runga-Kutta Gill algorithm was used for the numerical integration of the set of ordinary differential equations. Before the decoupling techniques has been applied, liquid level Y_1 depended on U_1 (flow rate) and liquid temperature Y_2 depended on U_1 and U_2 (heater input). After it had been applied Y_1 depends only U_1 and Y_2 depends only U_2.

Development of Hirschorn's control law

In order to calculate a control law that induces linear input/ output, the behavior of a MIMO system was carried out using Decoupling and Linearization (Hirschor's) algorithm[3]. It helps to find a differential operator such that, when applied to the outputs. It will provide a set of algebraic expressions in 'x' and 'u' that gives solution to 'u'.

The control law allows controlling linear systems without having to impose any structural constraints on the closed-loop dynamics of the system[4] and[5]. Therefore, the control designer has the flexibility to adjust the parameters $\beta_{ik,}$ for fast closed-loop dynamics and desirable level of coupling.

From Kravaris and Soroush, if

$$\varsigma^{(k^*)} = m \qquad \ldots(5)$$

$$F_l(x) = constant,\ \ l = 0, ...k^* - 1$$

Then the systems eqn. 1 is input/output linearizable. Furthermore, given mx1matrice $\beta_{ik,}\ i = 0, ...m, k = 0, ...r_i - 1$

$m\,x(m-\varsigma^{(0)}), m\,x(m-\varsigma^{(1)}), ...m\,x(m-\varsigma^{(k^*-1)})$ matrices $\gamma_0, \gamma_{1,..........}\gamma_{k^*-1}$ and an m x m invertible matrix Γ.

The state feedback law as given in[6] is reproduced below:

$$U = \left[\Gamma L_g H^{(k^*)}(x)\right]^{-1} \left\{ V - \sum_{i=1}^{m} \sum_{k=0}^{r_i-1} \beta_{ik} L_f{}^k h_i(x) - \right.$$

$$\left. \sum_{l=0}^{k^*-1} \gamma_l [F_l \vdots I_m - \varsigma^{(1)}] E_l L_f H^{(l)}(x) - \Gamma L_f H^{(k^*)}(x) \right\} \quad ...(6)$$

Applying the linearising algorithm, the decoupling and linearization control law obtained from the state feedback eqn (3) from[2] is given as,

$$U_1 = s\left[V_1 - \delta_{10} x_1 + \frac{k}{s} x_1^{1/2} - \frac{k^2}{2s^2}\right] \quad ...(7)$$

$$U_2 = -\left(\frac{To - x_2}{x_1}\right) C_p \varsigma s\, x_1 \left[V_1 - \delta_{10} x_1 + \frac{k}{s} x_1^{1/2} - \frac{k^2}{2s^2}\right] * C_p \varsigma\, s\, x_1 [V_2 - \delta_{20} x_2] \quad ...(8)$$

Applying the procedure and data as given in[1] and substituting U_1 and U_2 in the eqn (3) and (4) the state equation is obtained in both decoupled and linearised forms. The resulting eqns (9) and (10) are in decoupled form.

$$\frac{dx_1}{dt} = V_1 - \zeta_{10} x_1 - \frac{k^2}{2s^2} \quad ....(9)$$

$$\frac{dx_2}{dt} = V_2 - \zeta_{20} x_2 \quad ...(10)$$

The advantage of using Hirschorn's algorithm is that the control law is less complex. In addition to that it also offers more dynamic feed flow rate of liquid U_1 and heat input rate U_2. The simulation results show that Hirschorn's algorithm has better effect.

Implementation of Kalman filter

Kalman Filter is a recursive predictive filter based on the use of state space techniques and recursive algorithms, i.e. only the estimated state from the previous time step and the current measurement are needed to compute the estimate of the current state**.** The Kalman filter operates by propagating the mean and covariance of the state through time. The notation $\hat{x}_{n|m}$ represents the estimate of the state vector 'X' at time n given observations till m.

The state of the filter is represented by two variables

- $\hat{x}_{k|k}$, a posteriori state estimate at time 'k'. The given observation is up to and including at time 'k'.
- $P_{k|k}$, a posteriori error covariance matrix which measure the estimated accuracy of the state.

The Kalman filter has two distinct phases, prediction and correction[7]. The prediction phase uses the state estimate from the previous time step to produce an estimate of the state at the current time step. This predicted state estimate is also known as the apriori state estimate because, it is an estimate of the state at the current time step and it does not include observation information from the current time step. In the correction phase, the current apriori prediction is combined with current observation information to refine the state estimate. This improved estimate is termed the a posteriori state estimate. The block diagram in Figure 2 represents the state estimator.

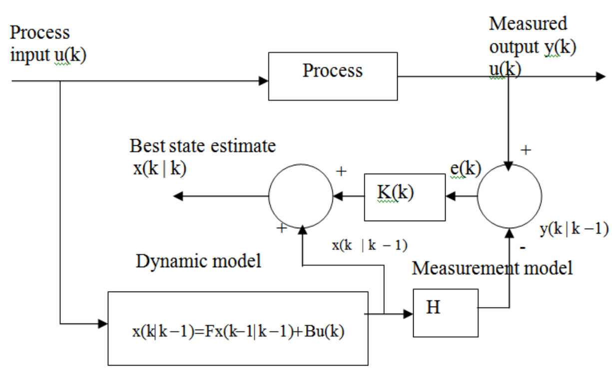

Fig. 2: Block diagram of Kalman filter

Typically, the two phases alternate, with the prediction advancing the state until the next scheduled observation, and the correction incorporating the observation. However, this is not necessary, if an observation is unavailable for some reason, the update may be skipped and multiple prediction steps can be performed. Consider a linear time invariant discrete system given by the following equations,

$$X_k = FX_{k-1} + Bu_k + W_k \qquad \ldots(11)$$

$$Z_k = HX_k + V_k \qquad \ldots(12)$$

where, F is the state transition matrix,

B is the control input matrix,

W_k is the process noise with zero mean multivariate normal distribution having covariance Q_k.

H is the observation matrix,

V_k is the observation noise which is zero mean Gaussian white noise having covariance R_k.

U_k is the control input.

(a) Prediction (Time update) Equations

Predicted state estimate

$$\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1} + Bu_k \qquad \ldots(13)$$

Predicted estimate covariance

$$P_{k|k-1} = FP_{k-1|k-1} F_k^T + Q_k \qquad \ldots(14)$$

(b) Correction (Measurement update) Equations

Innovation or measurement residual

$$\hat{y}_k = Z_k - H\hat{X}_{k|k-1} \qquad \ldots(15)$$

Innovation (or residual) covariance

$$S_k = H P_{k|k-1} H^T + R_k \quad \ldots(16)$$

Optimal Kalman gain

$$K_k = P_{k|k-1} H^T S_k^{-1} \quad \ldots(17)$$

Updated (a posteriori) state estimate

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k \hat{y}_k \quad \ldots(18)$$

Updated (a posteriori) estimate covariance

$$P_{k|k} = (I - K_k H) P_{k|k-1} \quad \ldots(19)$$

Extended Kalman filter

As we know the real systems that are inspiration for all these estimators like KF are governed by nonlinear functions. So we always need the advanced version of the filters that are basically designed for linear filters. Similarly, it is said that in estimation theory, the EKF is the nonlinear version of the Kalman filter[8]. This nonlinear filter linearizes about the current mean and covariance. At one time, the EKF might have been considered the standard in the nonlinear state estimation navigation systems and GPS.

Formulation

In the EKF, the state transition and observation state space models may not be linear functions of the state but might be many non-linear functions.

Equation $X_k = f(X_{k-1}, u_{k-1}) + W_{k-1}$

$$Z_k = h(X_k) + V_k$$

Where W_k and V_k are the process and observation noise. Mean multivariate Gaussian noise with covariance Q_k and R_k, respectively[9].

The functions ‘f’ and ‘h’ use the previous estimate and help in computing the predicted state and the predicted state is used to calculate the predicted measurement. However, ‘f’ and ‘h’ cannot be used to the covariance directly. So a matrix of partial derivatives (the Jacobian) computation is required. At each time step with the help of current predicted states the Jacobian is calculated. These matrices are used in the KF equations. This process actually linearizes the non-linear function around the present estimate.

Predict and update equations

(a) Prediction equations

Predicted state

$$\hat{X}_{k|k-1} = f(\hat{X}_{k-1|k-1}, u_{k-1})$$

Predicted estimate covariance

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^{T} + Q_{k-1}$$

Correction Equations

Innovation or measurement residual

$$\hat{Y}_k = Z_k - h(\hat{X}_{k|k-1})$$

Innovation (or residual) covariance

$$S_k = H_k P_{k|k-1} H_k^{T} + R_k$$

Optimal kalman gain

$$K_k = P_{k|k-1} H_k^{T} S_k^{-1}$$

Updated state estimate

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k Y_k$$

Updated estimate covariance

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

Measurement model H_k

where the state transition and observation matrices are defined to be the following Jacobians.

$$F_{k-1} = \frac{\partial f}{\partial x} \mid \hat{X}_{k-1|k-1}, u_{k-1}$$

$$H_k = \frac{\partial h}{\partial x} \mid \hat{X}_{k|k-1}$$

Simulation results and discussion

Utilizing the model given by[2], decoupling and linearization algorithm was designed. At first the simulation was carried out without decoupling. Figure 4 shows the output response of the level and temperature when the set point of the level and temperature were changed from 1 to 20 cm and 1 to 10°C, respectively. When sudden disturbance was introduced at 200 sec in level, it affected the temperature process due to interaction.

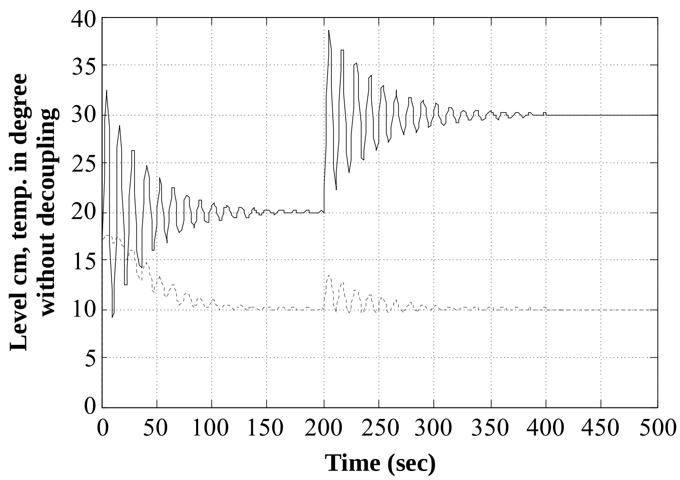

Fig. 4: Out put response for the step change in the level without decoupling with PI controllers

Simulation was carried out after applying Hirchorn's algorithm with external PI controller as shown in Fig. 5. The sudden disturbance introduced at 200 sec in level did not affect the temperature process. It can be seen from the Figure 4 that under the influence of the controller, ISE is improved.

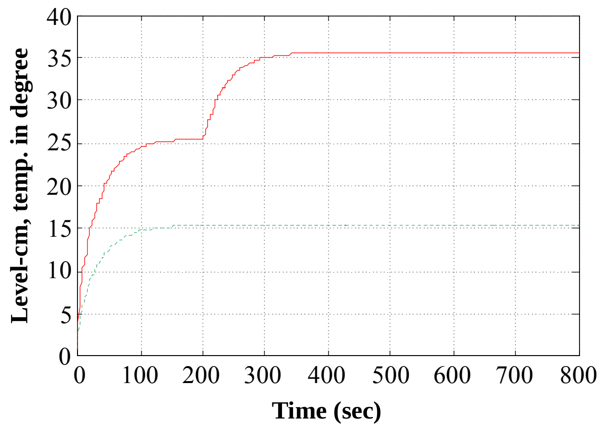

Fig. 5: Output response for the step change in the level with decoupling with PI controllers

Figure 6 illustrates the level tracking error between plant and Kalman filter. Kalman filter was designed for the level process of the state space model. Actual level output of y and observer level output ŷ were obtained directly from simulation model of the plant and state estimation error was calculated. Using KF the level error varied between -2 to +2 cm. So 10% error occurred as mentioned in Table 1.

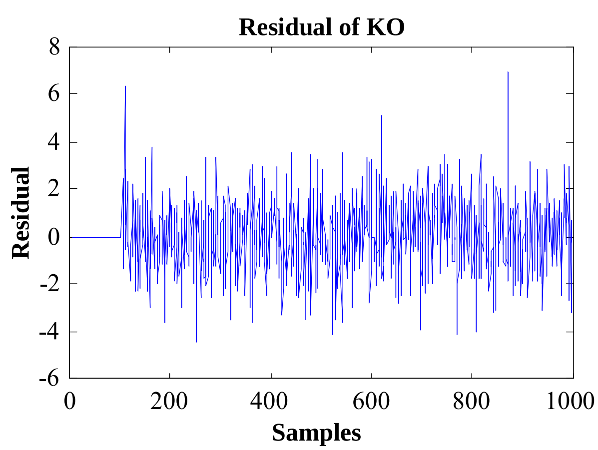

Fig. 6: Estimated error of the plant and Kalman filter for level parameter

Fig. 7 illustrates the temperature tracking errors for plant and Kalman filter. Kalman filter was designed for the temperature process of the state space model. Actual temperature output of y and observer temperature output ŷ were obtained directly from the simulation model of the plant and state estimation error was calculated. Using KF the temperature error varied between -2 to +2ºC. So 15% error occurred as mentioned in Table 1.

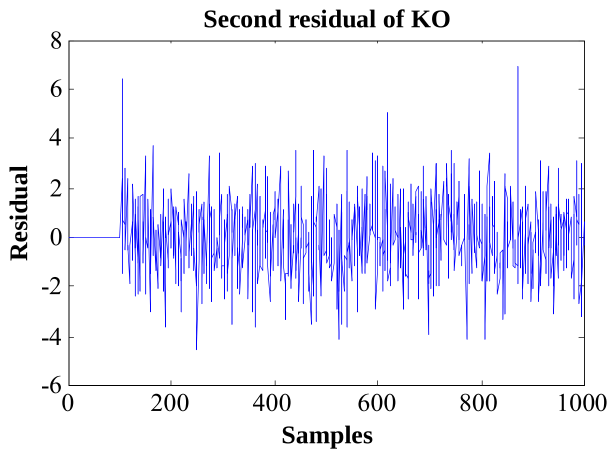

Fig. 7: Estimated error of the plant and Kalman filter for temperature parameter

Fig. 8 illustrates the level tracking errors for plant and EKF. EKF was designed for the level process of the state space model. Actual level output of y and observer level output $\hat{y}$ were obtained directly from the simulation model of the plant and state estimation error was calculated. Using EKF the level error varied between +0.02 to -0.02 cm. So the filter output follows the system output and only 0.1% error occurred.

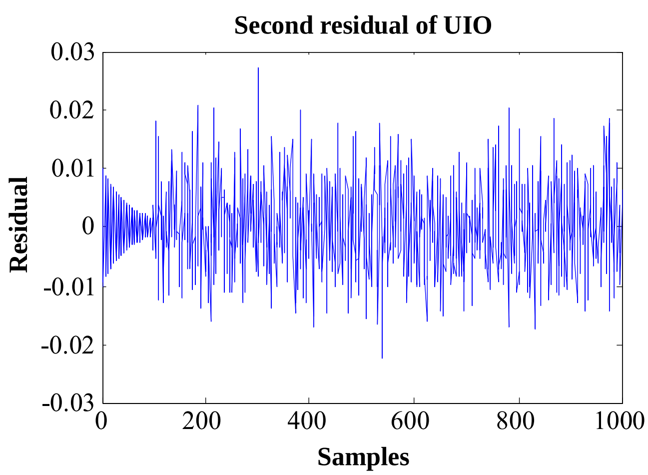

Fig. 8: Estimated error of the plant and EKF for level parameter

Fig. 9 illustrates the temperature tracking errors for plant and EKF. EKF was designed for the temperature parameter of the state space model. Actual temperature output of y and observer temperature output $\hat{y}$ were obtained directly from simulation model of the plant and the state estimation error was calculated. Using EKF the temperature error varied between +0.02 to -0.02 cm. So the filter output follows the system output and only 0.1% error occurred.

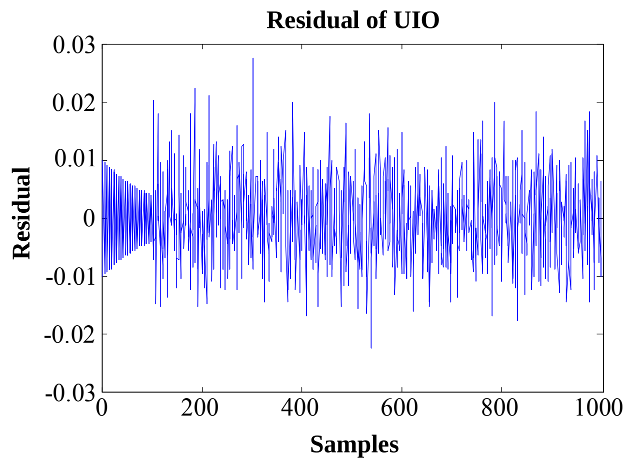

Fig. 9: Estimated error of the plant and EKF for temperature parameter

Table 1: Comparison of observer performance

Filter type	Level parameter error	Temp parameter error
KF	10%	15%
EKF	0.1%	0.15%

CONCLUSION

The decoupling linearization algorithm was applied to a nonlinear MIMO interacting thermal process. The simulation results had shown that even if the processes are non-linear and interactive a satisfactory control performance could be obtained. Then Kalmanfilter and Extended Kalman filter was designed to estimate the system parameters like level and temperature for a non-interacting linear MIMO system. Results of these simulations are presented in Table 1. Performance of EKF was found to be better. The obtained outputs for EKF give less error when compared to Kalman filter.

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