



CHEMICAL REACTION AND SORET EFFECTS ON CASSON MHD FLUID FLOW OVER A VERTICAL PLATE

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ABSTRACT

In this work, chemical reaction and solet effects on casson MHD fluid flow over a vertical plate with heat source/sink is examined. This problem is solved numerically using the perturbation technique for the velocity, the temperature and the concentration species. The skin friction, nusselt number and sherwood number are also obtained and are shown in tabular form. The effects of various physical parameters like as chemical reaction parameter, Soret parameter, radiation parameter, heat parameter, casson parameter, schmidt number, grashof number, prandtl number, hartmann parameter, and modified grashof numbers have been discussed in detailed.

Key words: Casson fluid, Soret effect, MHD, Chemical reaction.

INTRODUCTION

In recent years, MHD non-Newtonian flow fluid in the presence of solet and dufour effects has received more attention. In fact, this article has direct significant application in related to non-Newtonian fluids like as casson fluids (human blood, honey etc.), nano fluids and power law fluids etc. In industrial environment, non-Newtonian flow fluids play a vital role. There are many transport processes occurring in nature due to temperature and chemical differences. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is a working medium. The phenomena on chemical reaction and the influences of solet and dufour in various spheres analysed and presented by the authors¹⁻¹⁶. In this work, chemical reaction and Soret effects on casson MHD fluid flow over a vertical

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plate with heat source/sink has been examined. This problem is solved numerically using the perturbation technique for the velocity, the temperature and the concentration species. The skin friction, Nusselt number and Sherwood number are also obtained and are shown in tabular form. The effects of various physical parameters like as chemical reaction parameter, Soret parameter, radiation parameter, heat parameter, casson parameter, Schmidt number, Grashof number, Prandtl number, Hartmann parameter, and modified Grashof number has been discussed in detailed. The results are made in this article are good agreement with previous work¹⁷.

Formulation of the problem

MHD Casson fluid of incompressible, viscous, electrically-conducting fluid over a vertical plate moving with constant velocity with radiation and chemical reaction in the presence of Soret effect is considered. The rheological equation of state for an isotropic and incompressible flow of Casson fluid^{3,4} is –

$$\tau_{ij} = \begin{cases} \left(\mu_B + P_y / \sqrt{2\pi} \right) 2e_{ij}, & \pi > \pi_c \\ \left(\mu_B + P_y / \sqrt{2\pi_c} \right) 2e_{ij}, & \pi < \pi_c \end{cases}$$

Where μ_B is plastic dynamic viscosity, P_y is yield stress, π_c is critical value of π , and π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j)th component of deformation rate. The x-axis is taken along the plate in the vertical upward direction and the y-axis is taken normal to the plate. The surface temperature of the plate oscillates with small amplitude about a non-uniform mean temperature. The fluid is assumed to have constant properties except for the influence of the density variations with temperature and concentration, which are considered only in the body force term. The temperature of the plate oscillates with little amplitude about a non-uniform temperature. By usual Boussinesq's approximation, the flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u'}{\partial y'^2} + g\beta^* (C' - C'_\infty) - \frac{\sigma}{\rho} B_0^2 u' + g\beta (T' - T'_\infty) \quad \dots(1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q'_r}{\partial y'} + \frac{Q_0}{\rho C_p} (T' - T'_\infty) \quad \dots(2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_r (C' - C'_\infty) + S_0 \frac{\partial^2 T'}{\partial y'^2} \quad \dots(3)$$

Equations (1), (2) and (3) refers Momentum equation, energy equation and species equation, respectively. Where u is the velocity of the fluid, β is Casson parameter, Q_0 is the heat source/sink parameter, D is the molecular diffusivity, k is thermal conductivity, C is mass concentration, t is time, ν is the kinematics viscosity, g is the gravitational constant, β and β^* are the thermal expansions of fluid and concentration, T is temperature of fluid, ρ is density, c_p is the specific heat capacity at constant pressure, y is distance, q_r is the radiative flux, β_0 is the magnetic field, kr_0 is the chemical reaction rate constant. R.H.S. of equation (1), second term is thermal concentration effect, third term is magnetic effect, fourth term is thermal buoyancy effect. R.H.S. of equation (2) second term is thermal radiation flux and third term is thermal radiation. R.H.S. of equation (3), second term is chemical reaction and third term solet (Thermo diffusion) effect. Under the above assumptions the physical variables are functions of y and t .

The boundary conditions are:

$$\begin{aligned} u' &= U, T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{i\omega t}, C' = C'_w + (C'_w - T'_\infty)e^{i\omega t} \quad \text{at } y=0 \\ u' &\rightarrow 0, T' \rightarrow 0, C' \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad \dots(4)$$

Introducing the dimensionless quantities –

$$\begin{aligned} u &= \frac{u'}{U}, y = \frac{y'U}{\nu}, t = \frac{t'U^2}{\nu}, Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U^3}, Gc = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U^3} \\ Q &= \frac{Q_0\nu}{\rho c_p U^2}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, M = \frac{\sigma\beta_0^2\nu}{\rho U^2}, R = \frac{16a\sigma^* \nu^2 T'^3_\infty}{U^2 \rho c_p}, \\ \mu &= \nu\rho, Pr = \frac{\mu c_p}{k}, Sc = \frac{\nu}{D}, kr = \frac{kr_0\nu}{U^2} \end{aligned} \quad \dots(5)$$

The thermal radiation flux gradient may be expressed as follows –

$$-\frac{\partial q_r}{\partial y'} = 4a\sigma^*(T'^4_\infty - T'^4) \quad \dots(6)$$

Considering the temperature difference by assumption within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is attained by expanding in T'^4 taylor's series about T'_∞ and ignoring higher orders terms.

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \quad \dots(7)$$

Substituting the dimensionless variables (5) into (1) to (3) and using equations (6) and (7), reduce to the following dimensionless form.

$$\begin{aligned}\frac{\partial u}{\partial t} &= \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - Mu + Gr\theta + GcC \\ \frac{\partial \theta}{\partial t} &= \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - (R - Q)\theta \\ \frac{\partial C}{\partial t} &= \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - K_r C + Sr \frac{\partial^2 \theta}{\partial y^2}\end{aligned}\quad \dots(8)$$

The corresponding boundary conditions of (4) in dimensionless form are –

$$\begin{aligned}u = 1, \theta = 1 + \epsilon e^{iwt}, C = 1 + \epsilon e^{iwt} & \quad \text{at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 & \quad \text{as } y \rightarrow \infty\end{aligned}\quad \dots(9)$$

Method of solution

Equation (8) represent a set of partial differential equations that cannot be solved in closed form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as –

$$\begin{aligned}u &= u_0 + \epsilon e^{iwt} u_1 + O(\epsilon^2) + \dots \\ \theta &= \theta_0 + \epsilon e^{iwt} \theta_1 + O(\epsilon^2) + \dots \\ C &= C_0 + \epsilon e^{iwt} C_1 + O(\epsilon^2) + \dots\end{aligned}\quad \dots(10)$$

Where $u_0(y)$, $u_1(y)$, $\theta_0(y)$, $\theta_1(y)$, $C_0(y)$ and $C_1(y)$ have to be determined.

$$\begin{aligned}\left(1 + \frac{1}{\beta}\right) u''_0 - M u_0 &= -Gr\theta_0 - GcC_0 \\ A_4 u''_1 - (M + iw)u_1 &= -Gr\theta_1 - GcC_1 \\ \frac{1}{Pr} \theta''_0 &= (R - Q)\theta_0 \\ \frac{1}{Pr} \theta''_1 &= (R - Q + iw)\theta_1 \\ C''_0 - Sck_r C_0 &= -ScSr\theta''_0 \\ C''_1 - Sc(k_r + iw)C_1 &= -ScSr\theta''_1\end{aligned}\quad \dots(11)$$

All primes denote differentiation with respect to y .

The boundary conditions are –

$$\begin{aligned} u_0 &= 1, \theta_0 = 1, C_0 = 1 & \text{at } y = 0, \\ u_1 &= 0, \theta_1 = 1, C_1 = 1 & \text{at } y = 0, \\ u_0 &\rightarrow 0, \theta_0 \rightarrow 0, C_0 \rightarrow 0 & \text{as } y \rightarrow \infty, \\ u_1 &\rightarrow 0, \theta_1 \rightarrow 0, C_1 \rightarrow 0 & \text{as } y \rightarrow \infty. \end{aligned} \quad \dots(12)$$

Solving the system (11) subject to the boundary conditions (12),

We obtain –

$$\begin{aligned} u_0 &= B_1 e^{-\sqrt{A_5} y} + \left(\frac{A_6}{A_1 - A_5} \right) e^{-\sqrt{A_1} y} + \left(\frac{A_7}{Sckr - A_5} \right) e^{-\sqrt{Sckr} y} \\ u_1 &= B_2 e^{-\sqrt{A_8} y} + \left(\frac{A_6}{A_2 - A_8} \right) e^{-\sqrt{A_2} y} + \left(\frac{A_7 g_2}{A_3 - A_8} \right) e^{-\sqrt{A_3} y} + \left(\frac{A_7 g_1}{A_2 - A_8} \right) e^{-\sqrt{A_2} y} \\ \theta_0 &= e^{-\sqrt{A_1} y}, \theta_1 = e^{-\sqrt{A_2} y}, C_0 = e^{-\sqrt{Sckr} y}, C_1 = g_2 e^{-\sqrt{A_3} y} + g_1 e^{-\sqrt{A_2} y}. \end{aligned} \quad \dots(13)$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer becomes,

$$\begin{aligned} u &= u_0 + \varepsilon e^{i\omega t} u_1, \\ \theta &= \theta_0 + \varepsilon e^{i\omega t} \theta_1, \\ C &= C_0 + \varepsilon e^{i\omega t} C_1 \end{aligned} \quad \dots(14)$$

The skin friction (C_f), Nusselt number (Nu) and Sherwood number (Sh) are obtained from equation (14) when differentiated at $y = 0$.

$$\begin{aligned} C_f &= \left(-B_1 \sqrt{A_5} + \frac{A_6}{A_1 - A_5} \sqrt{A_1} + \frac{A_7}{Sckr - A_5} \sqrt{Sckr} \right) + \\ &\varepsilon e^{i\omega t} \left(-B_2 \sqrt{A_8} - \frac{A_6}{A_2 - A_8} \sqrt{A_2} - \frac{A_7 g_2}{A_3 - A_8} \sqrt{A_3} - \frac{A_7 g_1}{A_2 - A_8} \sqrt{A_2} \right), \\ Nu &= \sqrt{A_1} + \varepsilon e^{i\omega t} \sqrt{A_2}, \\ Sh &= \sqrt{Sckr} + \varepsilon e^{i\omega t} \sqrt{A_3} \end{aligned} \quad \dots(15)$$

RESULTS AND DISCUSSION

Casson MHD flow over a vertical plate with chemical reaction parameter and solet parameter have been formulated and analysed analytically. Only three computations are performed for variation of the velocity with thermal Grashof number (Gr), variation of the temperature with heat source/sink parameter (Q), variation of the concentration with Schmidt number (Sc). The dimensional governing equations are solved by two term perturbation technique in this article with $Pr = 2$, $G_r = 2.0$, $G_c = 2.0$, $\varepsilon = 0.01$, $M = 0.5$, $t = 1.0$, $Sc = 2.0$, $kr = 0.5$, $R = 0.2$, $Q = 0.1$, $\omega = 1.0$, $Sr = 1$. All graphs therefore correspond to these unless specifically indicated on the appropriate graph.

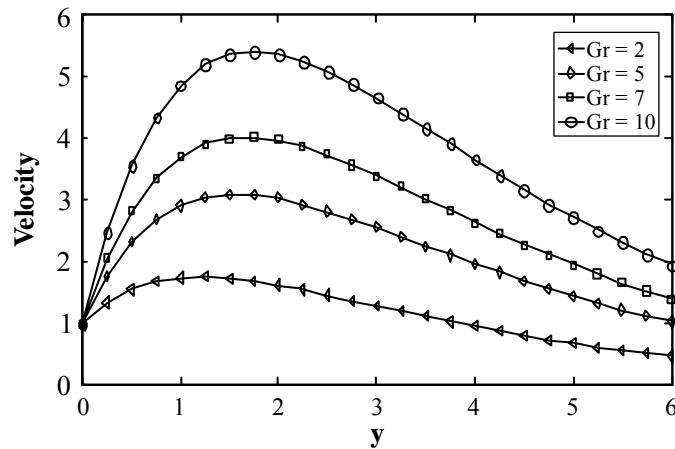


Fig. 1: Variation of the velocity with Hartmann number

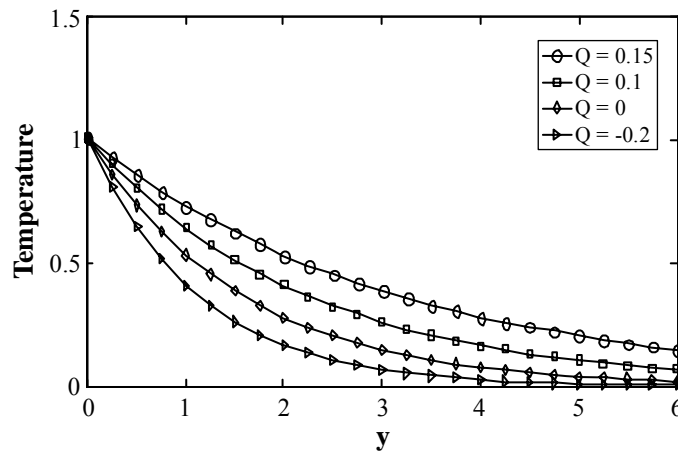


Fig. 2: Variation of the temperature with heat source/sink parameter

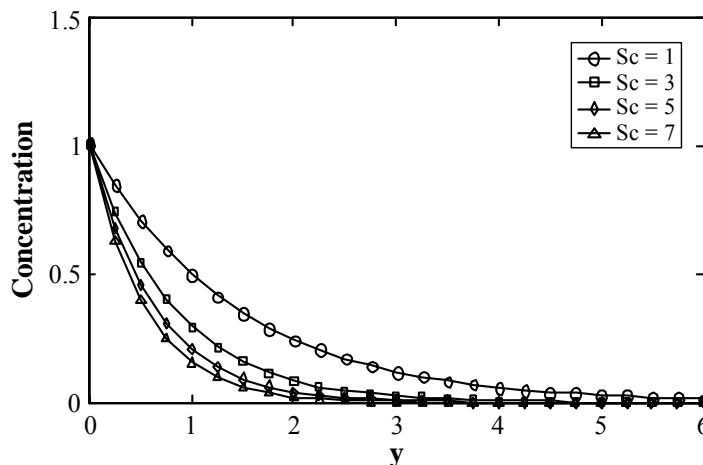


Fig. 3: Variation of the concentration with Schmidt number

The influence of thermal Grashof number in velocity is shown in Fig. 1. It is recognised that velocity increases with increasing of the thermal Grashof number. Fig. 2 shows the influence of heat source/sink parameter on the temperature. It is cleared that temperature is increased, when Q is increased. The influence of Sc on the concentration is illustrated in Fig. 3. The concentration decreases as the schmidt increases.

Table 1: Variation of the the Skin-friction coefficient, Nusselt number and Sherwood number

Gr	Gc	kr	C_f	Nu	Sh
4	2	0.5	2.0801	0.8945	1.0001
6	2	0.5	2.9861	0.8945	1.0001
2	4	0.5	2.0194	0.8945	1.0001
2	6	0.5	2.8648	0.8945	1.0001
2	2	1	0.9982	0.8945	1.4144
2	2	2	0.8461	0.8945	2.0002

Table 1 shows the effects of thermal Grashof number, mass Grashof number, chemical reaction parameter with $Pr = 2$, $G_r = 2.0$, $G_C = 2.0$, $\epsilon = 0.01$, $M = 0.5$, $t = 1.0$, $Sc = 2.0$, $kr = 0.5$, $R = 0.2$, $Q = 0.1$, $\omega = 1.0$, $Sr = 1$.

CONCLUSION

In this article, we have examined chemical reaction, radiation and Soret effects on Casson MHD fluid flow over a vertical plate with heat source/sink. This problem is solved numerically using the perturbation technique for the velocity, the temperature and the concentration species. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamics force. The flow is raised due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number. It is observed that the thermal Grashof number influence the velocity field almost in the boundary layer when compared to far away from the plate. The skin friction, Nusselt number and Sherwood number are also obtained and are shown in tabular form. The skin-friction is increased with the effect of casson parameter, at the plate. As perturbation parameter increases, the Skin-friction coefficient, Nusselt number and Sherwood number decreases at the plate and hence it will not be discussed any further, due to brevity.

ACKNOWLEDGEMENT

We express our sincere gratitude to Dr. K. S. Balamurugan, Associate Professor in R.V.R & J.C., Guntur, for his kind co-operation and suggestions in completion of this article.

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Revised : 13.01.2016

Accepted : 15.01.2016